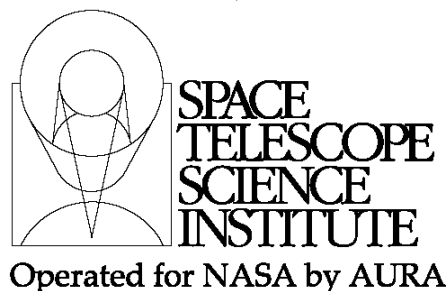




TECHNICAL REPORT



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|---------------------------------------|--|
| Title: NIRCAM Optimal Readout Modes | Doc #: JWST-STScI-001721, SM-12 Date: 13 April 2009 Rev: - |
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1.0 Abstract

NIRCAM readout scheme allows for 110 combinations of the two parameters “readout pattern” and “number of groups”. I estimate how these combinations affect the effective signal-to-noise ratio in the readout-noise-limited regime, first without and then with the effect of cosmic rays. It turns out that only 47 combinations are “optimal”, i.e. achieve the highest signal-to-noise ratio given their effective (clock) exposure time. In most cases, ramps made of groups averaging a large value of individual frames are preferable. The MEDIUM2 and DEEP2 readout patterns, performing long integrations skipping most of the frames, should never be used, at least in readout-noise-limited conditions.

2.0 Introduction

Raw JWST science images will be strongly affected by cosmic-rays (CRs). It is estimated that for HgCdTe detectors the number of CR events expected at L2 is of the order of 10 protons/cm²/s, with each event affecting on average 8 pixels (M. Rieke, private communication). This is sufficient to affect about 25% of the NIRCAM pixels in a 1000 s exposure.

To mitigate this problem, JWST detectors will be operated exploiting the fact that they can be read out non-destructively (for a justification see e.g. “Detector Readout Strategies” of the STScI-JWST-OPS-002010 Operation Concept Document). Besides achieving, for any given integration time, lower readout noise than that obtained sampling the signal once at the beginning and at the end of the integration, this provides a convenient way to isolate and remove cosmic ray impacts: if non-destructive samples of the signal are frequently taken during the integration (“up-the-ramp sampling”, or “multiaccum” mode in the NICMOS jargon) it may be possible to detect a CR event as a sudden jump of the signal between adjacent samples. One can therefore cut the integration in segments, before and after the cosmic ray event(s), extracting and combining in an optimal way the signal rates to recover the pixel (Regan et al. 2007; Robberto 2008), albeit with some degradation of the signal-to-noise ratio. The average loss of signal-to-noise depends on the assumed fluency of CRs, on the readout pattern,

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and on the number of frames (possibly coadded into groups, in the case e.g. of NIRCAM) considered, i.e., for a given readout pattern, on the integration time.

In this report I provide a statistical analysis of the signal-to-noise ratio, limited to the readout-noise-limited case (faint objects), achieved for all 9 types of NIRCAM readout patterns. I will refer to any given combinations of readout pattern and number of groups as a “mode”. Out the 110 possible NIRCAM modes, I will then select those that provide the highest signal-to-noise ratio for increasing readout time, in the presence of cosmic rays. These combinations may represent the optimal modes for NIRCAM operations, at least in readout-noise-limited regime.

3.0 Up-the-Ramp sampling

In the case of Up-the-Ramp sampling, we will assume (consistently with what planned is for NIRCAM) to have integrations, or “ramps”, sampled uniformly in time every dt seconds. In the case of NIRCAM it is $dt = 10.6$ seconds, for full array readout. Sets of adjacent samples can be either coadded into **groups** or ignored. The number of samples coadded into a group is given by N_{frames} (in this document we will use the terms “**sample**” and “**frame**” as equivalent, the first one being more appropriate when one talks about single pixels and the second one when one refers to the full array). For NIRCAM, the maximum value of frames in a group is $N_{frames} = 8$. If $N_{frames} = 1$ the group is made by an individual frame. Between the groups, there are N_{skip} samples that are ignored, i.e. not saved in the spacecraft data recorder and transmitted to the ground. Skipping frames provides a delay time between groups. Any given combination of N_{frames} and N_{skip} defines a **readout pattern**.

An integration ramp is therefore sampled using a certain readout pattern and N_{group} groups. Table 1 summarizes the 9 NIRCAM readout patterns, whereas Figure 1 illustrates the structure of a ramp in the case of the SHALLOW2 readout pattern with 3 groups. In this case it is $N_{frames} = 2$, $N_{skip} = 3$ and $N_{group} = 3$. For a more complete description of the NIRCAM readout patterns, see e.g. Section 4.2 “Multiaccum Readout Mode” of the NIRCAM Operation Concept Document (JWST-OPS-002843).

Table 1. Parameters of the 9 NIRCAM readout patterns

| <i>Readout pattern</i> | <i>Maximum Ngroup</i> | <i>Nframes</i> | <i>Nskip</i> |
|-------------------------------|------------------------------|-----------------------|---------------------|
| DEEP 8 | 20 | 8 | 12 |
| DEEP2 | 20 | 2 | 18 |
| MEDIUM8 | 10 | 8 | 2 |
| MEDIUM2 | 10 | 2 | 8 |
| SHALLOW4 | 10 | 4 | 1 |
| SHALLOW2 | 10 | 2 | 3 |

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| <i>Readout pattern</i> | <i>Maximum Ngroup</i> | <i>Nframes</i> | <i>Nskip</i> |
|------------------------|-----------------------|----------------|--------------|
| BRIGHT2 | 10 | 2 | 1 |
| BRIGHT1 | 10 | 1 | 1 |
| RAPID | 10 | 1 | 0 |

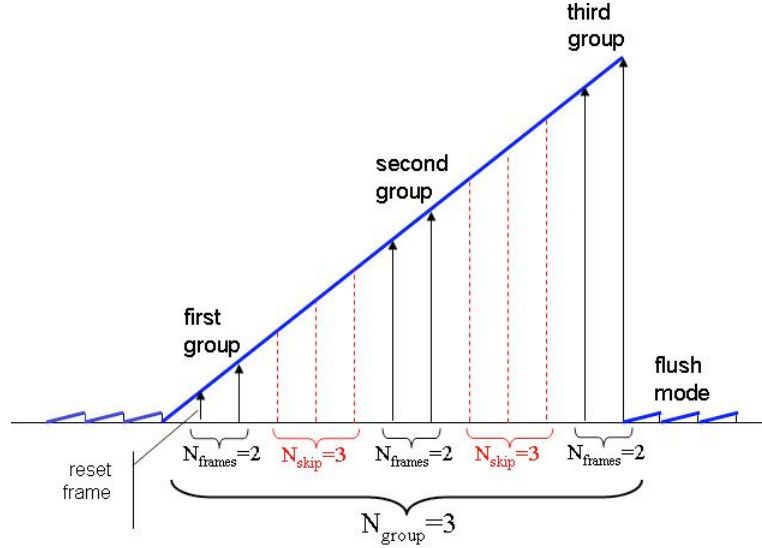


Figure 1 In MULTIACCUM mode, each pixel is reset and then non-destructively sampled many times during an integration, or ramp. Sampling occurs in ‘groups’, each composed by N_{frames} sent to the Focal Plane Array Processor for coadding. Other N_{skip} frames between group are ignored. At the end, the detector is reset and returns in flush mode. The very first frame of the ramp, the “RESET” frame, is also indicated.

In order to increase the dynamic range, the very first frame of the very first group (the so-called “RESET” frame, see Figure 1) is treated in two different ways: 1) it is averaged with the other frames of the first group, and 2) it is stored separately. The difference between the average of the first group and the RESET frame allows extracting the signal of bright sources without waiting for the end of the second group, avoiding possible saturation. The readout noise of the single RESET frame will dominate the readout noise of the differential image, but this is not an issue if the goal is to recover the brightest stars in the field. Keeping a separate RESET frame may also allow to clean the first average from possible “reset anomalies” often present in the first frame after a detector reset. For our analysis, we shall not consider the individual RESET frame, dealing therefore only with sequences containing an equal number of frames averaged into equally spaced groups, according to Table 1.

Averaging the frames of a group has an implication on the effective readout time, which now falls in the middle of each group. At the end of ramp, the time needed to read the first half of the first group and the second half of the last group appear as an overhead in the estimate of the attained signal-to-noise ratio.

Assuming a ramp composed by N_{group} groups obtained at the (average) time t_i , each providing a value y_i , the signal rate (i.e. the slope of the ramp) is given by the expression:

$$s = \frac{\sum_{i=1}^{N_{group}} (t_i - \bar{t})(y_i - \bar{y})}{\sum_{i=1}^{N_{group}} (t_i - \bar{t})^2} \quad (1)$$

where \bar{t} and \bar{y} refer to the values averaged over the N_{frame} frames contained in a group. The slope s is known with an uncertainty (variance) given by:

$$\sigma_s^2 = \frac{\sigma_V^2}{\sum_{i=1}^{N_{group}} t_i^2 - N\bar{t}^2} \quad (2)$$

In the case of readout-noise-limited regime, one assumes $\sigma_V = \sigma_{RON}$, the detector readout noise. In this case signal is present but very small, i.e. with negligible effect on the noise. This is the most demanding case in astronomy, neglecting other source of errors like 1/f noise.

Usually (Fowler and Garnett, 1993), one assumes uniform sampling of the signal:

$$t_i = i \cdot dt \quad (3)$$

which, at the end of the ramp, when $i = N_{group}$, becomes $t_{N_{group}} = N_{group} \cdot dt = T_{int}$. In this case, Equation (2) gives:

$$\sigma_s^2 = \frac{12\sigma_{RON}^2}{dt^2 \cdot N_{group} (N_{group} - 1)(N_{group} + 1)} \quad (4)$$

for the variance of the slope. The same expression multiplied by T_{int} provides the variance of the signal. The signal-to-noise ratio obtained accumulating the signal for T_{int} seconds is therefore given by:

$$SNR_{line-fit} = \frac{s \cdot T_{int}}{\frac{\sqrt{12\sigma_{RON}^2}}{dt \sqrt{N_{group} (N_{group} - 1)(N_{group} + 1)}} T_{int}} \quad (5)$$

which simplifies into the expression:

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$$SNR_{line-fit} = \frac{s \cdot dt}{\sigma_{RON}} \sqrt{\frac{N_{group}(N_{group}-1)(N_{group}+1)}{12}} \quad (6)$$

This relation shows that in readout-noise limited regime the signal-to-noise depends only on 1) flux rate between adjacent samples ($s \cdot dt$), 2) the readout noise of each sample (σ_{RON}), and 3) the total number of groups (N_{group}). Writing explicitly the integration time one has:

$$SNR_{line-fit} = \frac{FT_{int}}{\sigma_{RON}} \sqrt{\frac{(N_{group}-1)(N_{group}+1)}{12N_{group}}}. \quad (7)$$

Apparently, the derivation of Equation (7) depends on the assumption made in Equation (3) that the first sample has accumulated a valid signal of duration dt . This is not always the case. For example, if we refer to Figure 1, we see that the time interval between groups (timed at the central point of each group) is $5dt$, whereas the distance between the last reset and the very first group is $1.5dt$. The discrepancy between the very first group and the others disappears only when all groups are made by single frames and no frames are skipped, i.e. (in the case of NIRCcam) when one is taking a RAPID ramp. On the other hand, the timing of the very first sample after the reset does not affect these equations. This is derived explicitly in Appendix A.

The values of $SNR_{line-fit}$ obtained for the 9 NIRCcam readout pattern and number of groups, i.e. for the 110 NIRCcam modes, are listed in Table 1 and shown in Figure 2. We assumed a readout noise $\sigma_{RON} = 15 \text{ e}^-$ and a signal $F = 0.01 \text{ e/s}$ are. The same values are reported in Table 2, normalized to the highest value reached at the end of a DEEP8 ramp. Due to the “quantization” of the readout times in multiples of 10.6 seconds, there are certain values of the integration time which are common to different modes. In these cases, one will achieve different signal-to-noise ratios depending on the readout pattern. Also, it is possible that the signal-to-noise ratio achieved in a certain mode is lower than that achieved in another mode having a shorter integration time. Our results show that readout patterns with lower values of N_{read} (red symbols in Figure 2) generally provide lower signal-to-noise ratio than the readout patterns with higher values of N_{read} : this is not surprising, since the former have lower effective readout noise due to the group averaging. The latter, on the other hand, have been introduced because they provide a better treatment of the cosmic rays, which up to now have been neglected. In the next section we will therefore expand our analysis to include the case with cosmic rays, and see how this affects our results.

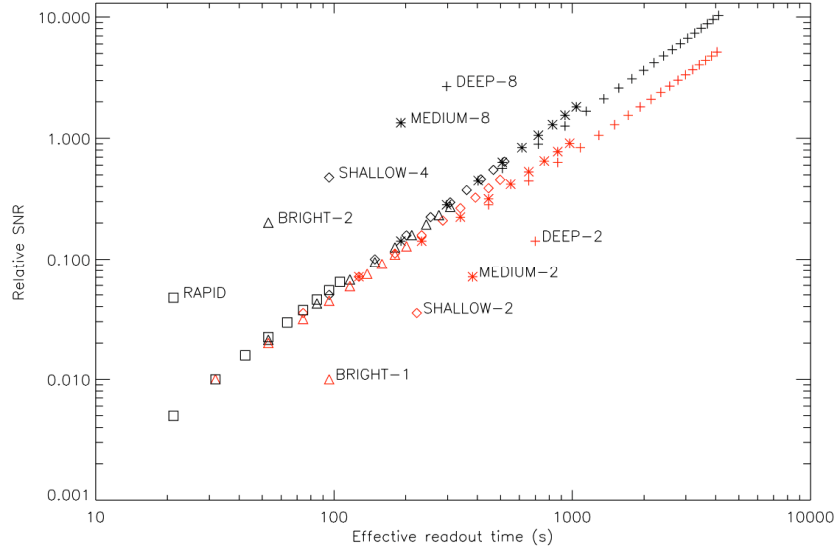


Figure 2 Signal-to-noise-ratio reached for the various NIRCcam readout modes

4.0 Effect of Cosmic Rays

The main advantage of Up-the-Ramp sampling sequences is that they can be screened for cosmic rays, which can be removed allowing for longer integration times and therefore high signal-to-noise in the readout-noise limited regime.

The effect of Cosmic- Rays in Up-the-Ramp sampling has been discussed by Offenberget al. (PASP, 2000) in the case of ramps with $N_{frames} = 1$. We will build on their treatment, adding the necessary adjustments needed when $N_{frames} \neq 1$.

We can consider four cases: case 0) has no cosmic-rays in the ramp, case 1) has ramps hit by 1 cosmic ray, case 2) has ramps hit by two cosmic rays and case 3) has ramps hit by three or more cosmic rays. The average signal rate, s_{eff} , is therefore given by

$$s_{eff} = \frac{s_0 P_0 w_0 + s_1 P_1 w_1 + s_2 P_2 w_2 + s_{3+} P_{3+} w_{3+}}{P_0 w_0 + P_1 w_1 + P_2 w_2 + P_{3+} w_{3+}} \quad (8)$$

where s_i are the rates, P_i the probabilities, and w_i are the weights relative to the various cases. The corresponding effective variance is given by the formula:

$$V_{eff}^2 = \sum_{i=0}^{3+} \left(\frac{\partial s_{eff}}{\partial s_i P_i} \right)^2 V(s_i P_i) \quad (9)$$

which becomes:

$$V_{eff} = \frac{V_0 P_0 w_0^2 + V_1 P_1 w_1^2 + V_2 P_2 w_2^2 + V_{3+} P_{3+} w_{3+}^2}{(w_0 P_0 + w_1 P_1 + w_2 P_2 + w_{3+} P_{3+})^2} \quad (10)$$

where we have indicated with V_i the variances of the signal in the various cases.

The signal to noise ratio, originally given by

$$SNR_0 = \frac{FT_{int}}{\sqrt{V_0}} \quad (11)$$

for a ramp without cosmic rays, must also be adjusted to account for the presence of cosmic rays, becoming:

$$SNR_{eff} = \frac{FT_{int}}{\sqrt{V_{eff}}} \quad (12)$$

As the weights are the inverse of the variance ($w_i = 1/V_i$), Eq. (10) becomes:

$$V_{eff} = \frac{\frac{P_0}{V_0} + \frac{P_1}{V_1} + \frac{P_2}{V_2} + \frac{P_{3+}}{V_{3+}}}{\left(\frac{P_0}{V_0} + \frac{P_1}{V_1} + \frac{P_2}{V_2} + \frac{P_{3+}}{V_{3+}}\right)^2} = \frac{1}{\frac{P_0}{V_0} + \frac{P_1}{V_1} + \frac{P_2}{V_2} + \frac{P_{3+}}{V_{3+}}} \quad (13)$$

$$\begin{aligned} SNR_{eff} &= FT_{int} \sqrt{\frac{P_0}{V_0} + \frac{P_1}{V_1} + \frac{P_2}{V_2} + \frac{P_{3+}}{V_{3+}}} \\ &= \frac{FT_{int}}{\sqrt{V_0}} \sqrt{P_0 + \frac{P_1}{V_1/V_0} + \frac{P_2}{V_2/V_0} + \frac{P_{3+}}{V_{3+}/V_0}} \end{aligned} \quad (14)$$

The probability P_i of a cosmic rays impact on a ramp is governed by the binomial distribution. The probability P_0 that a ramp has of surviving without a cosmic ray hit is related to the probability of surviving a time interval between frames, P_{dt} , by the relation $P_0 = P_{dt}^N$. This follows from the normal distribution: given N adjacent time intervals, the probability of having M intervals with a CR hit is given by:

$$P(M, N) = \binom{N}{M} P_{dt}^M (1 - P_{dt})^{N-M} \quad (15)$$

which becomes, in the case $M = 0$,

$$P(0, N) = P_{dt}^N \quad (16)$$

Accordingly, the probability of being hit once is given by

$$P_1 = P(1, N) = N \cdot P_{dt} \cdot (1 - P_{dt})^{N-1}, \quad (17)$$

and the probability of being hit twice is given by

$$P_2 = P(2, N) = \frac{N \cdot (N-1)}{2} P_{dt}^2 (1 - P_{dt})^{N-2} \quad (18)$$

We will arbitrarily assume that when a pixel is hit by 3 or more cosmic rays during an integration, it cannot be recovered. This simplifies our calculations, as we can impose:

$$P_{3+} = P(3+, N) = 1 - P_0 - P_1 - P_2 \quad (19)$$

Having calculated the variance V_0 in Section 2.0, our problem here is to calculate the variances V_1 and V_2 , whereas for V_3 we will simply assume the variance to be infinite. For any particular ramp, the variance V_1 depends on the frame interval in which the cosmic ray event occurs. For example, a cosmic ray hit between the reset and the first sample has no impact (unless the cosmic rays event saturates the pixel, a possibility we shall not consider in this study). A cosmic ray hit between the first and the second sample shrinks all ramps composed by more than 2 samples, reducing the number of samples by 1. A cosmic ray hit between the second and the third sample breaks the ramps in two pieces, and so on. The variance will become larger and larger for cosmic rays hitting closer to the middle and, of course, for multiple CR events. Equation (13) tells us how to combine variances. In the case of a single cosmic ray hit splitting the ramp in two semi-ramps with variances V_{pre} and V_{post} , the optimally averaged ramp has a variance

$$V_1 = \frac{1}{\frac{1}{V_{pre}} + \frac{1}{V_{post}}} \quad (20)$$

(having set the probabilities equal to 1). According to Equation (7), the variance of a ramp composed by N samples is proportional to $1 / N(N+1)(N-1)$ (we shall use N instead of N_{group} hereafter for short). Therefore, if the cosmic ray event occurs after the sample N_i , on has

$$V_1 = \frac{K}{N_i(N_i+1)(N_i-1) + (N-N_i)(N-N_i+1)(N-N_i-1)} \quad (21)$$

where the constant of proportionality can be set by using the expression for V_0 , obtaining

$$V_{1,i}(N) = V_0 \frac{N(N+1)(N-1)}{N_i(N_i+1)(N_i-1) + (N-N_i)(N-N_i+1)(N-N_i-1)} \quad (22)$$

If we assume that the cosmic ray events are randomly distributed over time, we can calculate the expectation values for all values $1 \dots N_i \dots N$:

$$V_1(N) = \frac{1}{N} \sum_{N_i=0}^{N-1} V_1(N_i) \quad (23)$$

where the index runs from 0 (a CR hit before the first sample), to N-1 (a CR hit before the last sample). Let's validate this expression by analyzing a couple of cases:

1. with one sample, there is of course no ramp. Both Numerator and Denominator in Eq. (22) are 0.
2. with two samples and one cosmic ray, the CR may hit either before the first sample or between the first and the second one. As shown in Table 3, the if the CR hits before the first sample we still have a valid ramp, with the same variance we would have obtained in the case of no CR hit. This happens 50% of the cases. For the remaining 50% of the cases the CR hit occurs between the two samples and the full ramp is lost. We may assume that in this case the variance is infinite. The effective variance can therefore be calculated by the relation:

$$V_1(2) = \frac{1}{\frac{P_{pre}}{V_{pre}} + \frac{P_{post}}{V_{post}}} = \frac{1}{\frac{0.5}{1} + \frac{0.5}{\infty}} = 2 \quad (24)$$

Table 2. Case of 1 cosmic ray hit in a ramp of two samples

| i=0 | | i=1 | | N=2: $M(M-1)(N+1)=6$ | | |
|-----------|--------|-----------|--------|-----------------------------------|--|-----------|
| dt | sample | dt | sample | $V_{pre}/V_0 = N_i(N_i-1)(N_i+1)$ | $V_{post}/V_0 = (N-N_i)(N-N_i-1)(N-N_i+1)$ | V_i/V_0 |
| CR @ i=0 | ok | integrate | ok | 0 | 6 | 1 |
| integrate | lost | CR @ i=1 | lost | 0 | 0 | INDEF |

3. with three samples, one has the situation described in Table 3:

Table 3. Case of 1 cosmic ray hit in a ramp of 3 samples

| i=0 | | i=1 | | i=2 | | N=3: $V_0 = M(M-1)(N+1)=24$ | | |
|-----------|--------|-----------|--------|-----------|--------|-----------------------------------|--|-----------|
| dt | sample | dt | sample | dt | sample | $V_{pre}/V_0 = N_i(N_i-1)(N_i+1)$ | $V_{post}/V_0 = (N-N_i)(N-N_i-1)(N-N_i+1)$ | V_i/V_0 |
| CR @ i=0 | ok | integrate | ok | integrate | ok | 0 | 24 | 1 |
| integrate | lost | CR @ i=1 | ok | integrate | ok | 0 | 6 | 4 |
| integrate | ok | integrate | ok | CR @ i=2 | lost | 6 | 0 | 4 |

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and the combined signal has a variance $V_1(3) = (1 + 4 + 4)/3 = 3$.

In the case of a ramp with N samples and 2 cosmic rays, Equation (22) can be immediately modified into the following expression:

$$V_2(N) = V_0 \frac{\sum_{N_j=1}^{N-1} \sum_{N_i=1}^{N_j-1} \frac{N(N+1)(N-1)}{N_i(N_i+1)(N_i-1) + (N_j - N_i)(N_j - N_i + 1)(N_j - N_i - 1) + (N - N_j)(N - N_j + 1)(N - N_j - 1)}}{\sum_{N_j=1}^{N-1} \sum_{N_i=1}^{N_j-1} 1} \quad (25)$$

The case $N=3$ with 2 cosmic rays is explicitly derived in the Table 4. Also in this case we have 1/3 of the probability of having a null ramp, if the two cosmic rays happen in the second and third interval. The variance is therefore

$$V_2(3) = \frac{1}{\frac{P_{pre}}{V_{pre}} + \frac{P_{post}}{V_{post}}} = \frac{1}{\frac{0.667}{4} + \frac{0.333}{\infty}} = 6 \quad (26)$$

Table 4. Case of 2 cosmic ray in a ramp of 3 samples

| i=0 | | i=1 | | i=2 | | N=3: $V_0=N(N-1)(N+1)=24$ | |
|-----------|--------|-----------|--------|-----------|--------|---------------------------|-------|
| dt | sample | dt | sample | dt | sample | Sum(Vi) | Vi/Vo |
| CR @ i=0 | lost | CR @ i=1 | ok | integrate | ok | 6 | 4 |
| CR @ i=0 | ok | integrate | ok | CR @ i=2 | ok | 6 | 4 |
| integrate | lost | CR @ i=1 | lost | CR @ i=2 | lost | 0 | INDEF |

In the case $N=4$ with 2 cosmic rays (Table 5) the average variance can be simply estimated from the average of the V_i/V_0 values, obtaining $V_2(4) = 6.667$

Table 5. Case of 2 cosmic ray hit in a ramp of 4 samples

| i=0 | | i=1 | | i=2 | | i=3 | | N=3: $V_0=N(N-1)(N+1)=60$ | |
|-----------|--------|-----------|--------|-----------|--------|-----------|--------|---------------------------|-------|
| dt | sample | dt | sample | dt | sample | dt | sample | Sum(Vi) | Vi/Vo |
| CR @ i=0 | lost | CR @ i=1 | ok | integrate | ok | integrate | ok | 24 | 2.5 |
| CR @ i=0 | ok | integrate | ok | CR @ i=2 | ok | integrate | ok | 2x6=12 | 5 |
| CR @ i=0 | ok | integrate | ok | integrate | ok | CR @ i=3 | lost | 24 | 2.5 |
| integrate | lost | CR @ i=1 | lost | CR @ i=2 | ok | integrate | ok | 6 | 10 |
| integrate | lost | CR @ i=1 | ok | integrate | ok | CR @ i=3 | lost | 6 | 10 |
| integrate | ok | integrate | ok | CR @ i=2 | lost | CR @ i=3 | lost | 6 | 10 |

Table 6 shows the expectation values for the relative variances in the cases with one and with two cosmic ray impacts.

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Table 6. Increase of variance in ramps with 1 or 2 CRs.

| CR before read | Vi/Vo | |
|----------------|-------|-------|
| | 1 CR | 2 CR |
| 1 | INDEF | INDEF |
| 2 | 2 | INDEF |
| 3 | 3 | 6.000 |
| 4 | 2.75 | 6.667 |
| 5 | 2.6 | 5.700 |
| 6 | 2.54 | 5.227 |
| 7 | 2.505 | 4.975 |
| 8 | 2.483 | 4.824 |
| 9 | 2.469 | 4.723 |
| 10 | 2.459 | 4.651 |
| 11 | 2.452 | 4.598 |
| 12 | 2.446 | 4.557 |
| 13 | 2.442 | 4.524 |
| 14 | 2.439 | 4.498 |
| 15 | 2.436 | 4.477 |
| 16 | 2.434 | 4.459 |
| 17 | 2.432 | 4.443 |
| 18 | 2.431 | 4.430 |
| 19 | 2.429 | 4.419 |
| 20 | 2.428 | 4.409 |

At this point, we have all the ingredients needed to calculate the effective variance, Equation (13), and the effective signal-to-noise rate, Equation (14), for the various readout modes in the presence of cosmic rays.

In the case of NIRCAM, however, there is one extra step that has to be made: when $N_{group} \neq 1$, a cosmic ray event has a different impact on the effective variance of the ramp depending if it falls within the samples that are skipped or within the samples that are averaged. In the first case, the previous treatment applies. In the second case, the full group of averaged samples is lost (the possibility of recovery has been occasionally considered but not yet demonstrated, and we will neglect it). The loss of a group of samples (and of the following skipped samples), has the practical consequence of shortening the total number of samples of the ramp by 1 unit. Therefore, in the case $N_{group} \neq 1$, for a ramp of N groups with 1 cosmic ray hit we shall assume that the fraction of time spent skipping or averaging simply determines the probability of having to consider N or $N - 1$ groups.

$$V_1 = \frac{1}{\frac{N_{skip}}{N_{group} + N_{skip}} \frac{1}{V_1(N)} + \frac{N_{group}}{N_{group} + N_{skip}} \frac{1}{V_1(N-1)}} \quad (27)$$

where $V_1(N)$ is given by Equation (13).

If two cosmic rays hit the ramp, there are 3 possibilities:

1. both cosmic rays hit during the N_{skip} samples.
2. one cosmic ray hits during the N_{skip} samples and one during the N_{read} samples.
3. both cosmic rays hit during the N_{read} samples.

Equation (27) now becomes:

$$V_2 = \frac{1}{\left(\frac{N_{skip}}{N_{group} + N_{skip}} \right)^2 \frac{1}{V_2(N)} + 2 \left(\frac{N_{skip}}{N_{group} + N_{skip}} \right) \left(\frac{N_{group}}{N_{group} + N_{skip}} \right) \frac{1}{V_2(N-1)} + \left(\frac{N_{group}}{N_{group} + N_{skip}} \right)^2 \frac{1}{V_2(N-2)}} \quad (28)$$

The expression for the variance V_{eff} (Equation (13)) and of the signal-to-noise ratio (Equation (14)) are therefore modified accordingly. Inserted in Equation (12), they provide the values shown in Figure 3 and listed in Table 7, right column.

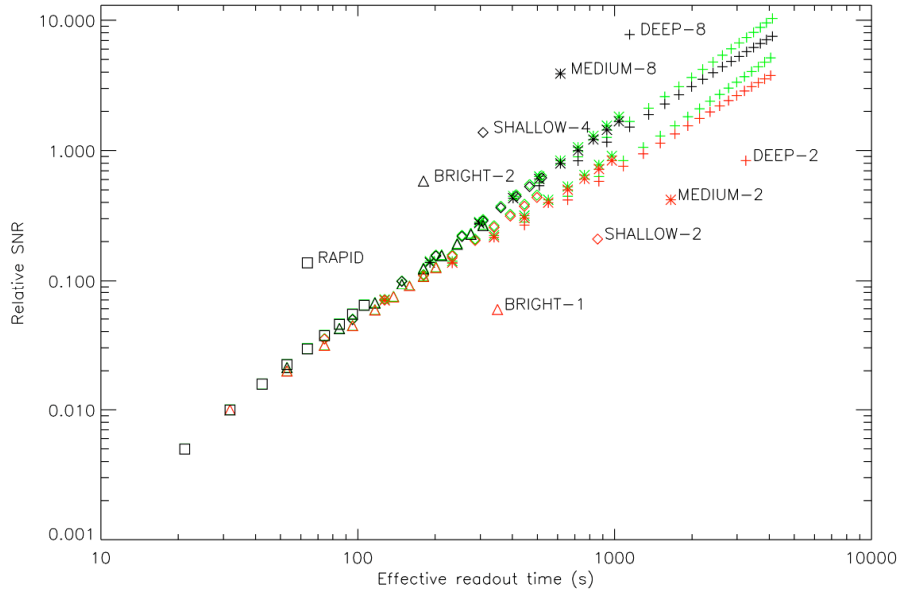


Figure 3 Signal-to-noise-ratio reached for the various NIRCcam readout ramps after considering the effect of cosmic rays. The green symbols refer to the corresponding cases without cosmic rays.

Table 7. Relative variance in modes with 1 or 2 CRs. Values are normalized to the SNR reached at the end of a DEEP8 readout pattern with no-CR (last row).

| <i>Tint</i> | <i>Ramp</i> | <i>read</i> | <i>SNR – no CR</i> | <i>SNR – with CR</i> |
|-------------|-------------|-------------|--------------------|----------------------|
| 10.6 | BRIGHT1 | 1 | 0 | 0 |
| 10.6 | RAPID | 1 | 0 | 0 |
| 21.2 | BRIGH2 | 1 | 0 | 0 |
| 21.2 | SHALLOW2 | 1 | 0 | 0 |
| 21.2 | RAPID | 2 | 0.00048 | 0.00048 |
| 21.2 | MEDIUM2 | 1 | 0 | 0 |
| 21.2 | DEEP2 | 1 | 0 | 0 |
| 31.8 | BRIGHT1 | 2 | 0.00097 | 0.00097 |
| 31.8 | RAPID | 3 | 0.00097 | 0.00097 |
| 42.4 | SHALLOW4 | 1 | 0 | 0 |
| 42.4 | RAPID | 4 | 0.00153 | 0.00153 |
| 53 | BRIGH2 | 2 | 0.00206 | 0.00204 |
| 53 | BRIGHT1 | 3 | 0.00194 | 0.00193 |
| 53 | RAPID | 5 | 0.00217 | 0.00216 |
| 63.6 | RAPID | 6 | 0.00287 | 0.00285 |
| 74.2 | RAPID | 7 | 0.00363 | 0.00361 |
| 74.2 | SHALLOW2 | 2 | 0.00343 | 0.00339 |
| 74.2 | BRIGHT1 | 4 | 0.00307 | 0.00304 |
| 84.8 | RAPID | 8 | 0.00444 | 0.00441 |
| 84.8 | DEEP8 | 1 | 0 | 0 |
| 84.8 | BRIGH2 | 3 | 0.00411 | 0.00408 |
| 84.8 | MEDIUM8 | 1 | 0 | 0 |
| 95.4 | RAPID | 9 | 0.00531 | 0.00527 |
| 95.4 | BRIGHT1 | 5 | 0.00434 | 0.0043 |
| 95.4 | SHALLOW4 | 2 | 0.00485 | 0.00479 |
| 106 | RAPID | 10 | 0.00623 | 0.00618 |
| 116.6 | BRIGHT1 | 6 | 0.00574 | 0.00568 |
| 116.6 | BRIGH2 | 4 | 0.0065 | 0.00643 |
| 127.2 | MEDIUM2 | 2 | 0.00686 | 0.00674 |
| 127.2 | SHALLOW2 | 3 | 0.00686 | 0.00677 |
| 137.8 | BRIGHT1 | 7 | 0.00725 | 0.00717 |
| 148.4 | BRIGH2 | 5 | 0.0092 | 0.00908 |
| 148.4 | SHALLOW4 | 3 | 0.00969 | 0.00959 |
| 159 | BRIGHT1 | 8 | 0.00889 | 0.00877 |
| 180.2 | SHALLOW2 | 4 | 0.01084 | 0.01065 |
| 180.2 | BRIGH2 | 6 | 0.01217 | 0.01198 |
| 180.2 | BRIGHT1 | 9 | 0.01062 | 0.01047 |
| 190.8 | MEDIUM8 | 2 | 0.01371 | 0.01338 |
| 201.4 | BRIGHT1 | 10 | 0.01245 | 0.01225 |

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To verify that this is the current version.

| <i>Tint</i> | <i>Ramp</i> | <i>read</i> | <i>SNR – no CR</i> | <i>SNR – with CR</i> |
|-------------|-------------|-------------|--------------------|----------------------|
| 201.4 | SHALLOW4 | 4 | 0.01533 | 0.01505 |
| 212 | BRIGH2 | 7 | 0.01539 | 0.01512 |
| 233.2 | SHALLOW2 | 5 | 0.01533 | 0.015 |
| 233.2 | DEEP2 | 2 | 0.01371 | 0.0133 |
| 233.2 | MEDIUM2 | 3 | 0.01371 | 0.01336 |
| 243.8 | BRIGH2 | 8 | 0.01885 | 0.01848 |
| 254.4 | SHALLOW4 | 5 | 0.02168 | 0.02121 |
| 275.6 | BRIGH2 | 9 | 0.02253 | 0.02204 |
| 286.2 | SHALLOW2 | 6 | 0.02028 | 0.01978 |
| 296.8 | DEEP8 | 2 | 0.02742 | 0.02639 |
| 296.8 | MEDIUM8 | 3 | 0.02742 | 0.02682 |
| 307.4 | BRIGH2 | 10 | 0.02642 | 0.02578 |
| 307.4 | SHALLOW4 | 6 | 0.02868 | 0.02796 |
| 339.2 | MEDIUM2 | 4 | 0.02168 | 0.02093 |
| 339.2 | SHALLOW2 | 7 | 0.02565 | 0.02492 |
| 360.4 | SHALLOW4 | 7 | 0.03627 | 0.03524 |
| 392.2 | SHALLOW2 | 8 | 0.03141 | 0.0304 |
| 402.8 | MEDIUM8 | 4 | 0.04336 | 0.04181 |
| 413.4 | SHALLOW4 | 8 | 0.04443 | 0.04299 |
| 445.2 | SHALLOW2 | 9 | 0.03755 | 0.0362 |
| 445.2 | MEDIUM2 | 5 | 0.03066 | 0.02939 |
| 445.2 | DEEP2 | 3 | 0.02742 | 0.026 |
| 466.4 | SHALLOW4 | 9 | 0.0531 | 0.05119 |
| 498.2 | SHALLOW2 | 10 | 0.04403 | 0.04228 |
| 508.8 | MEDIUM8 | 5 | 0.06131 | 0.05873 |
| 508.8 | DEEP8 | 3 | 0.05484 | 0.0522 |
| 519.4 | SHALLOW4 | 10 | 0.06226 | 0.05979 |
| 551.2 | MEDIUM2 | 6 | 0.04056 | 0.03859 |
| 614.8 | MEDIUM8 | 6 | 0.08111 | 0.07713 |
| 657.2 | MEDIUM2 | 7 | 0.0513 | 0.04844 |
| 657.2 | DEEP2 | 4 | 0.04336 | 0.04045 |
| 720.8 | MEDIUM8 | 7 | 0.1026 | 0.09684 |
| 720.8 | DEEP8 | 4 | 0.08671 | 0.08081 |
| 763.2 | MEDIUM2 | 8 | 0.06283 | 0.05888 |
| 826.8 | MEDIUM8 | 8 | 0.12566 | 0.11772 |
| 869.2 | MEDIUM2 | 9 | 0.07509 | 0.06983 |
| 869.2 | DEEP2 | 5 | 0.06131 | 0.0564 |
| 932.8 | MEDIUM8 | 9 | 0.15019 | 0.13963 |
| 932.8 | DEEP8 | 5 | 0.12263 | 0.11269 |
| 975.2 | MEDIUM2 | 10 | 0.08806 | 0.08125 |
| 1038.8 | MEDIUM8 | 10 | 0.17611 | 0.16247 |
| 1081.2 | DEEP2 | 6 | 0.08111 | 0.07351 |

Check with the JWST SOCCER Database at: <http://soccer.stsci.edu/DmsProdAgile/PLMServlet>
To verify that this is the current version.

| <i>Tint</i> | <i>Ramp</i> | <i>read</i> | <i>SNR – no CR</i> | <i>SNR – with CR</i> |
|-------------|-------------|-------------|--------------------|----------------------|
| 1144.8 | DEEP8 | 6 | 0.16222 | 0.14693 |
| 1293.2 | DEEP2 | 7 | 0.1026 | 0.0916 |
| 1356.8 | DEEP8 | 7 | 0.2052 | 0.18311 |
| 1505.2 | DEEP2 | 8 | 0.12566 | 0.1105 |
| 1568.8 | DEEP8 | 8 | 0.25131 | 0.22092 |
| 1717.2 | DEEP2 | 9 | 0.15019 | 0.13008 |
| 1780.8 | DEEP8 | 9 | 0.30038 | 0.26008 |
| 1929.2 | DEEP2 | 10 | 0.17611 | 0.15022 |
| 1992.8 | DEEP8 | 10 | 0.35222 | 0.30036 |
| 2141.2 | DEEP2 | 11 | 0.20336 | 0.17082 |
| 2204.8 | DEEP8 | 11 | 0.40671 | 0.34157 |
| 2353.2 | DEEP2 | 12 | 0.23186 | 0.1918 |
| 2416.8 | DEEP8 | 12 | 0.46372 | 0.38354 |
| 2565.2 | DEEP2 | 13 | 0.26157 | 0.21308 |
| 2628.8 | DEEP8 | 13 | 0.52315 | 0.4261 |
| 2777.2 | DEEP2 | 14 | 0.29245 | 0.23459 |
| 2840.8 | DEEP8 | 14 | 0.5849 | 0.46911 |
| 2989.2 | DEEP2 | 15 | 0.32444 | 0.25627 |
| 3052.8 | DEEP8 | 15 | 0.64889 | 0.51247 |
| 3201.2 | DEEP2 | 16 | 0.35752 | 0.27805 |
| 3264.8 | DEEP8 | 16 | 0.71504 | 0.55605 |
| 3413.2 | DEEP2 | 17 | 0.39164 | 0.29991 |
| 3476.8 | DEEP8 | 17 | 0.78328 | 0.59975 |
| 3625.2 | DEEP2 | 18 | 0.42678 | 0.32177 |
| 3688.8 | DEEP8 | 18 | 0.85356 | 0.64348 |
| 3837.2 | DEEP2 | 19 | 0.46291 | 0.34362 |
| 3900.8 | DEEP8 | 19 | 0.92582 | 0.68717 |
| 4049.2 | DEEP2 | 20 | 0.5 | 0.36539 |
| 4112.8 | DEEP8 | 20 | 1 | 0.73072 |

5.0 Optimal readout modes

With reference to the case with CRs, we can extract the list of “optimal modes” according to the following criteria:

- if a given readout time is reached with multiple modes, one selects the mode with the highest signal to noise ratio.
- if the signal-to-noise reached by a mode is lower than what can be reached by using a previous mode with shorter integration time, the mode is neglected.

This allows extracting the set of 47 modes listed in Table 8.

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To verify that this is the current version.

Table 8. Optimal modes vs. integration time, taking into account cosmic ray hits

| Best modes with cosmic rays | | | | | | |
|------------------------------------|-------------|------------------------|----------------|------------|-------------------------|---------------------------|
| <i>Nr.</i> | <i>Tint</i> | <i>Readout pattern</i> | <i>Ngroups</i> | <i>SNR</i> | <i>% of lost pixels</i> | <i>Equivalent RON (e)</i> |
| 1 | 21.2 | RAPID | 2 | 0.00048 | 0 | 42.485 |
| 2 | 31.8 | BRIGHT1 | 2 | 0.00097 | 0 | 31.907 |
| 3 | 42.4 | RAPID | 4 | 0.00153 | 0 | 26.927 |
| 4 | 53 | RAPID | 5 | 0.00216 | 0 | 23.817 |
| 5 | 63.6 | RAPID | 6 | 0.00285 | 0 | 21.622 |
| 6 | 74.2 | RAPID | 7 | 0.00361 | 0 | 19.958 |
| 7 | 84.8 | RAPID | 8 | 0.00441 | 0 | 18.638 |
| 8 | 95.4 | RAPID | 9 | 0.00527 | 0 | 17.557 |
| 9 | 106 | RAPID | 10 | 0.00618 | 0 | 16.649 |
| 10 | 116.6 | BRIGH2 | 4 | 0.00643 | 0 | 16.782 |
| 11 | 127.2 | SHALLOW2 | 3 | 0.00677 | 0 | 17.464 |
| 12 | 137.8 | BRIGHT1 | 7 | 0.00717 | 0.001 | 18.639 |
| 13 | 148.4 | SHALLOW4 | 3 | 0.00959 | 0 | 13.405 |
| 14 | 180.2 | BRIGH2 | 6 | 0.01198 | 0.001 | 14.157 |
| 15 | 190.8 | MEDIUM8 | 2 | 0.01338 | 0 | 11.147 |
| 16 | 201.4 | SHALLOW4 | 4 | 0.01505 | 0.001 | 11.954 |
| 17 | 212 | BRIGH2 | 7 | 0.01512 | 0.002 | 13.258 |
| 18 | 243.8 | BRIGH2 | 8 | 0.01848 | 0.003 | 12.519 |
| 19 | 254.4 | SHALLOW4 | 5 | 0.02121 | 0.002 | 10.906 |
| 20 | 275.6 | BRIGH2 | 9 | 0.02204 | 0.004 | 11.898 |
| 21 | 296.8 | MEDIUM8 | 3 | 0.02682 | 0.002 | 9.393 |
| 22 | 307.4 | SHALLOW4 | 6 | 0.02796 | 0.005 | 10.112 |
| 23 | 360.4 | SHALLOW4 | 7 | 0.03524 | 0.009 | 9.484 |
| 24 | 402.8 | MEDIUM8 | 4 | 0.04181 | 0.008 | 8.485 |
| 25 | 413.4 | SHALLOW4 | 8 | 0.04299 | 0.014 | 8.969 |
| 26 | 466.4 | SHALLOW4 | 9 | 0.05119 | 0.02 | 8.537 |
| 27 | 508.8 | MEDIUM8 | 5 | 0.05873 | 0.019 | 7.792 |
| 28 | 519.4 | SHALLOW4 | 10 | 0.05979 | 0.028 | 8.169 |
| 29 | 614.8 | MEDIUM8 | 6 | 0.07713 | 0.037 | 7.265 |
| 30 | 720.8 | MEDIUM8 | 7 | 0.09684 | 0.064 | 6.848 |
| 31 | 826.8 | MEDIUM8 | 8 | 0.11772 | 0.101 | 6.508 |
| 32 | 932.8 | MEDIUM8 | 9 | 0.13963 | 0.148 | 6.223 |
| 33 | 1038.8 | MEDIUM8 | 10 | 0.16247 | 0.207 | 5.981 |
| 34 | 1356.8 | DEEP8 | 7 | 0.18311 | 0.454 | 6.991 |
| 35 | 1568.8 | DEEP8 | 8 | 0.22092 | 0.698 | 6.726 |
| 36 | 1780.8 | DEEP8 | 9 | 0.26008 | 1.006 | 6.504 |
| 37 | 1992.8 | DEEP8 | 10 | 0.30036 | 1.381 | 6.316 |
| 38 | 2204.8 | DEEP8 | 11 | 0.34157 | 1.824 | 6.156 |

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| Best modes with cosmic rays | | | | | | |
|------------------------------------|-------------|------------------------|-----------------|------------|-------------------------|---------------------------|
| <i>Nr.</i> | <i>Tint</i> | <i>Readout pattern</i> | <i>Nggroups</i> | <i>SNR</i> | <i>% of lost pixels</i> | <i>Equivalent RON (e)</i> |
| 39 | 2416.8 | DEEP8 | 12 | 0.38354 | 2.336 | 6.019 |
| 40 | 2628.8 | DEEP8 | 13 | 0.4261 | 2.919 | 5.9 |
| 41 | 2840.8 | DEEP8 | 14 | 0.46911 | 3.57 | 5.798 |
| 42 | 3052.8 | DEEP8 | 15 | 0.51247 | 4.29 | 5.708 |
| 43 | 3264.8 | DEEP8 | 16 | 0.55605 | 5.075 | 5.631 |
| 44 | 3476.8 | DEEP8 | 17 | 0.59975 | 5.925 | 5.563 |
| 45 | 3688.8 | DEEP8 | 18 | 0.64348 | 6.837 | 5.505 |
| 46 | 3900.8 | DEEP8 | 19 | 0.68717 | 7.807 | 5.454 |
| 47 | 4112.8 | DEEP8 | 20 | 0.73072 | 8.834 | 5.411 |

Table 8 also list the fraction of pixels receiving 3 or more cosmic ray impacts, according to Equation (19), and therefore completely lost according to our definitions, and the equivalent readout noise, estimated from the effective signal-to-noise ratio achieved in the actual clock time. This therefore includes the overhead for detector readout time, and for this reason the shortest exposure in RAPID mode, with two samples, pays the largest price (the integration time is 10.6 seconds, but the effective clock time is 21.2 seconds, cutting in half the effective signal-to-noise ratio and therefore doubling the equivalent readout noise). The equivalent readout noise may represent a useful quantity in the estimate of the instrument sensitivity e.g. by an Exposure Time Calculator.

Finally, it is interesting to consider how the various readout patterns contribute to the list of optimal modes. As shown in Table 9, the distribution of the most “popular” readout patterns is strongly non-uniform. Four readout patterns (RAPID, SHALLOW4, MEDIUM8 and DEEP8) cover 82% of the cases. At the other extreme, BRIGHT1, SHALLOW2 cover only 3 integration times, and MEDIUM2 and DEEP2 never make it to the list of optimal modes. In general, the readout patterns with the larger number of N_{frames} provide higher signal-to-noise, even taking into account the cosmic ray events; it is clear that optimizing the readout pattern for lower readout noise provides overall better signal-to-noise ration than optimizing for optimal CR rejection. From this point of view, it could be interesting to see in a future study how much gain can be further reached by using for all readout patterns the limit case $N_{skip} = 0$.

Table 9 Frequency of readout pattern in the optimal modes

| RAMP | Nr. of entries |
|-------------|-----------------------|
| RAPID | 8 |
| BRIGHT1 | 2 |
| BRIGHT2 | 5 |
| SHALLOW2 | 1 |
| SHALLOW4 | 8 |
| MEDIUM2 | 0 |
| MEDIUM8 | 9 |
| DEEP2 | 0 |
| DEEP8 | 14 |

Figures 4 to 6 summarize our results. In Figure 4 we plot the integration times corresponding to the optimal modes. In Figure 5 we plot the relative signal-to-noise ratio in function of the integration time. In Figure 6 we plot the number of groups corresponding to the optimal modes in function of the integration time. This may be useful to estimate the data volume required by programs requiring different amounts of integration time.

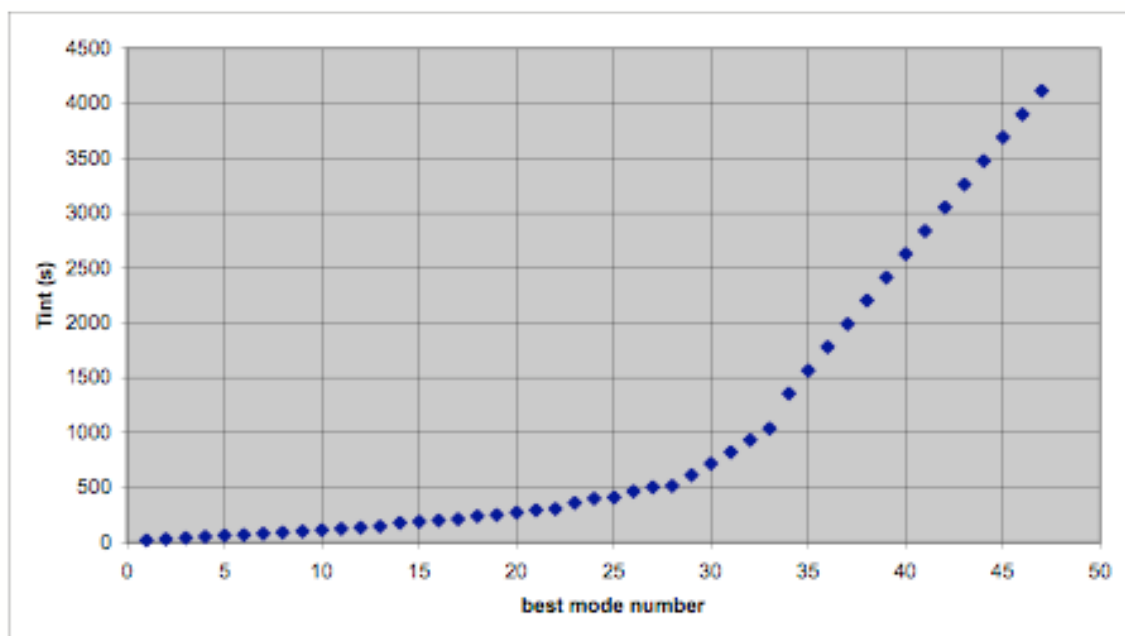


Figure 4 Integration times of the best mode number (listed in Table 8).

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To verify that this is the current version.

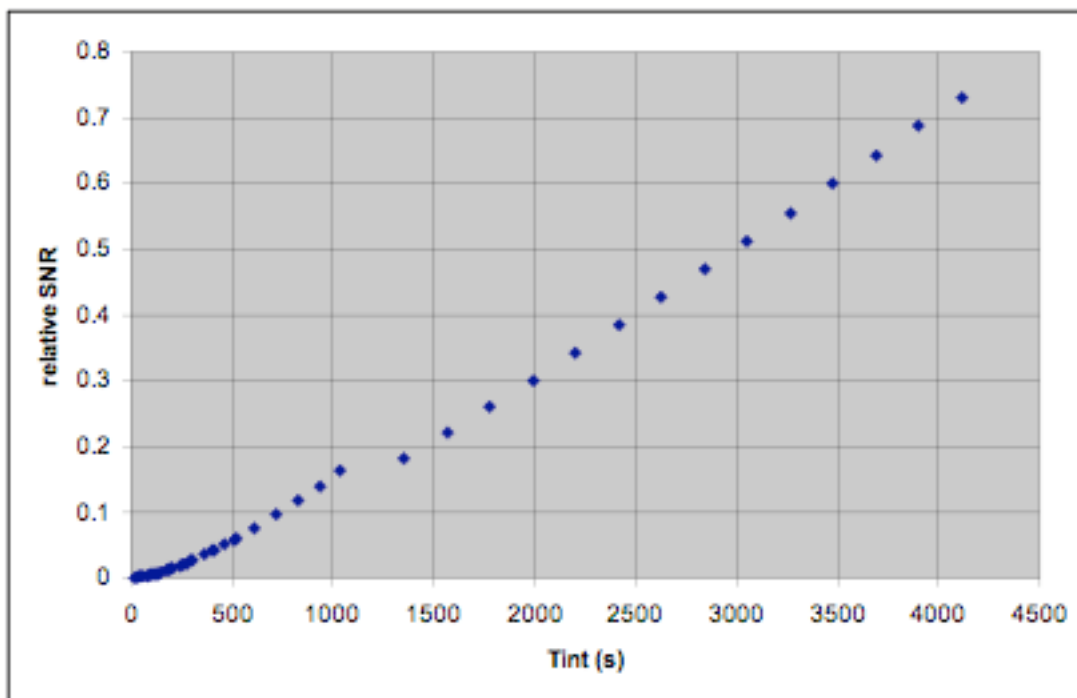


Figure 5 Integration times of the best mode number (listed in Table 8).

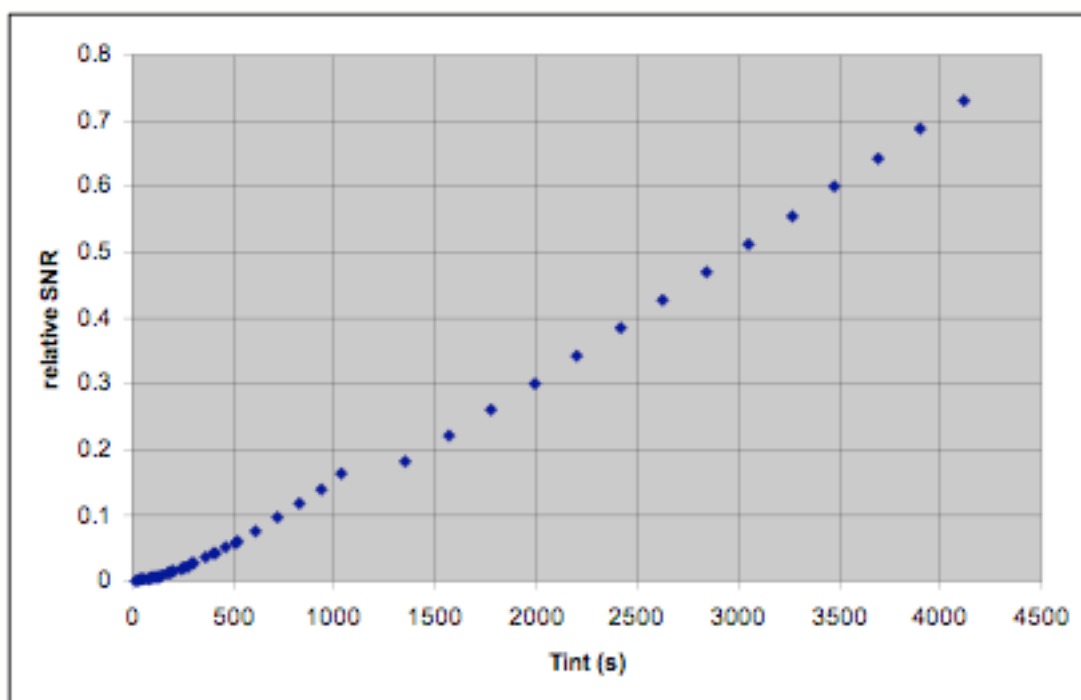


Figure 6 Relative signal-to-noise ratio reached by the optimal modes in function of the integration time.

Check with the JWST SOCCER Database at: <http://soccer.stsci.edu/DmsProdAgile/PLMServlet>
To verify that this is the current version.

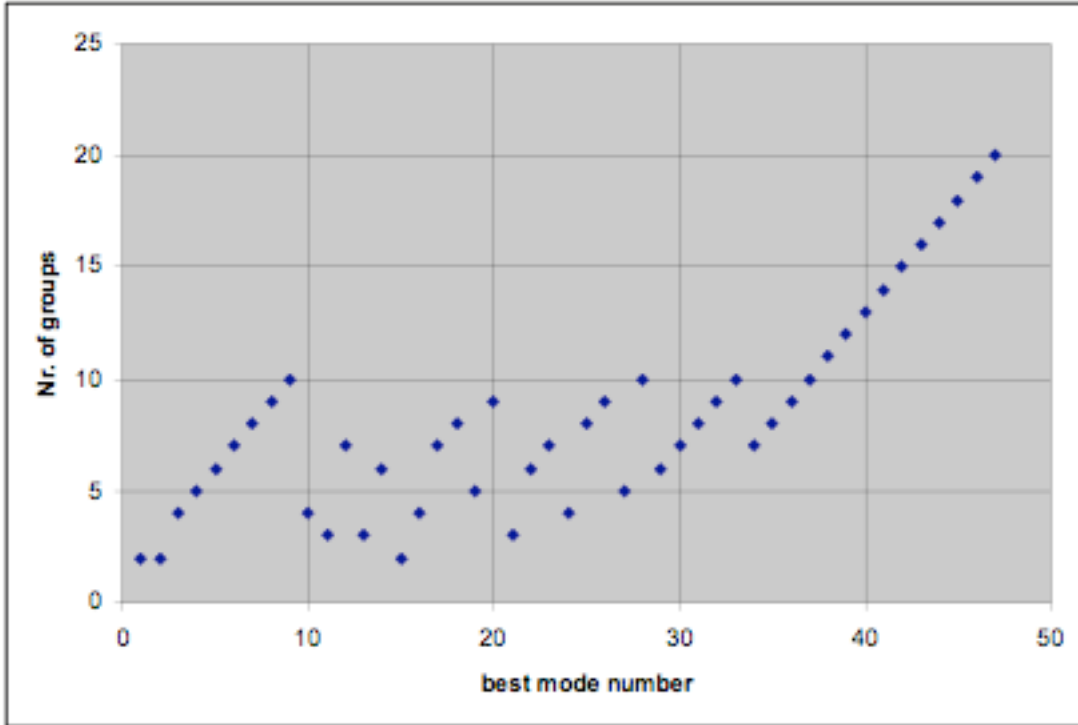


Figure 7 Number of groups needed by the optimal modes in function of the best mode number (listed in Table 8).

6.0 Conclusions

NIRCAM readout scheme allows for 110 modes, i.e. combinations of readout pattern and number of groups. However, not all of them are optimal for faint object imaging. I have estimated how the various combinations of parameters affect the effective signal-to-noise ratio in readout-noise-limited regime, including the effect of cosmic rays. It turns out that only 47 combinations are optimal, in the sense of achieving the highest signal-to-noise ratio given their corresponding amount of effective (clock) exposure times. In most cases having a large value of N_{frames} is preferable. The MEDIUM2 and DEEP2 ramps should never be used, at least in readout-noise-limited conditions.

These 47 combinations may provide the exposure times recommended by default by the APT for the faint-object case, i.e. after the observer has entered his exposure time the APT may choose the combination of readout pattern and number of groups corresponding to closest exposure time listed in Table 7. A future study will cover the case of subarrays, bright object imaging and coronagraphy.

7.0 References

Offenberg et al. 2001, “Validation of Up-the-Ramp Sampling with Cosmic Ray Rejection on IR Detectors”, PASP 113, 240

Regan, M., 2000, JWST-STScI-001212 “Optimum weighting of up-the-ramp readouts and how to handle cosmic rays”
Robberto M., 2008 “Optimal Strategy to fit MULTIACCUM ramps in the presence of cosmic rays”, JWST-STScI-001490

8.0 Appendix

Let us assume for convenience that the very first sample (or group of samples) occurs Δt seconds after the reset. We also label the very first sample with the index 0, leaving the other samples with an index running from 1 to N . We have now $N_{group} = N + 1$.

We can therefore replace Equation (3) for the integrated signal with the expression:

$$t_i = \Delta t + i \cdot dt \quad (29)$$

with $i = 0 \dots N$. To calculate the variance, we have now:

$$\sigma_m^2 = \frac{\sigma_y^2}{\sum_{i=0}^N t_i^2 - (N+1)\bar{t}^2} \quad (30)$$

where

$$\begin{aligned} \sum_{i=0}^N t_i^2 &= \sum_{i=0}^N (\Delta t + i dt)^2 \\ &= \sum_{i=0}^N (\Delta t^2 + 2i\Delta t dt + i^2 dt^2) \\ &= (N+1)\Delta t^2 + \sum_{i=1}^N (2i\Delta t dt + i^2 dt^2) \\ &= (N+1)\Delta t^2 + N(N+1)\Delta t dt + \frac{1}{6}N(2N^2 + 3N + 1)dt^2 \end{aligned} \quad (31)$$

and

$$\begin{aligned}
 \bar{t}^2 &= \left(\frac{\sum_{i=0}^N t_i}{N+1} \right)^2 \\
 &= \left(\frac{\sum_{i=0}^N (\Delta t + i dt)}{N+1} \right)^2 \\
 &= \left(\frac{(N+1)\Delta t + \frac{1}{2}N(N+1)dt}{N+1} \right)^2 \\
 &= \left(\Delta t + \frac{1}{2}Ndt \right)^2
 \end{aligned} \tag{32}$$

remembering the definitions of the power sums (Robberto 2007). Therefore:

$$\begin{aligned}
 \sigma_m &= \frac{\sigma_{RON}}{\sum_{i=0}^N t_i^2 - (N+1)\bar{t}^2} \\
 &= \frac{\sigma_{RON}}{(N+1)\Delta t^2 + N(N+1)\Delta t dt + \frac{1}{6}N(2N^2 + 3N + 1)dt^2 - (N+1)\left(\Delta t^2 + N\Delta t dt + \frac{1}{4}N^2 dt^2\right)} \tag{33} \\
 &= \frac{\sigma_{RON}}{\frac{1}{6}N\left(2N^2 + 3N + 1 - (N+1)\frac{1}{4}N^2\right)dt^2} \\
 &= \frac{12\sigma_{RON}}{dt^2 N(N+1)(N+2)}
 \end{aligned}$$

which is identical to Equation (4) considering that starting from index 0 we have one extra sample. Equation (5) thus remains unchanged.

The expression for the signal means that the RESET sample (or group of samples) performed after the physical reset of the pixel has to be ignored.

$$Signal = F \cdot dt \cdot N_{group} \tag{34}$$

where $IT=10.6$ s is the time needed to read the detector and $dt = (N_{frame} + N_{skip}) \cdot IT$ is the time between groups. The corresponding noise, assuming readout-noise limited regime, is

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$$Noise = \sqrt{\frac{12 N_{group}}{(N_{group} + 1)(N_{group} + 2)}} \sigma_{RON} \quad (35)$$

In our expressions, we are assuming that the first sample is number 0 and the last one is N_{group} . Therefore a ramp is actually composed by $N_{group} + 1$ samples. In practice, the first sample (sample 0) has $Signal = 0$ and $Noise = 0$ (there is no linear fit to a single point); the second sample (sample 1) has $Signal = F \cdot dt$ and $Noise = \sqrt{2} \sigma_{RON}$ (with two samples one has the double correlated sampling noise) and the third sample (sample 2) has $Signal = 2 \cdot F \cdot dt$ and $Noise = \sqrt{2} \sigma_{RON}$ (still the double correlated noise since the one degree of freedom is now lost for the estimate of the intercept). The signal-to-noise ratio at the end of a ramp is therefore:

$$\begin{aligned} SNR &= \frac{F \cdot dt \cdot N_{group}}{\sqrt{\frac{12 N_{group}}{(N_{group} + 1)(N_{group} + 2)}} \sigma_{RON}} \\ &= \frac{F \cdot dt \cdot N_{group}}{\sigma_{RON}} \sqrt{\frac{(N_{group} + 1)(N_{group} + 2)}{12}} \end{aligned} \quad (36)$$

valid for a ramp of $N_{group} + 1$ samples.