Maintaining JWST PSF quality

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22 CFR 125.4(b)2

Security

Newton Huygens Fourier Fraunhofer Born Sommerfeld Rayleigh Arly Fresnel Kirchhoff Fabry Perot de Broglie Heisenberg Poisson Fermat Lagrange Bessel Parseval Mach Zehnder Zernike
Math & pictures

If the math goes over your head…

don’t worry: pictures will follow

If the pictures bore you…

don’t worry, math will follow

There is still scope for physical intuition amongst all the equations of optical theory!!!
The telescope

- Primary ~ f/1.5 (by eye from the figure)
- ‘Cass’ f/16.7 (from the web)

- The Wavefront Sensor Camera
  - NIRCam short wave arm ~1-2.5 µm
  - 18 µm pixel H2RG
  - 32 mas/pixel
  - Some special optics & software
Hexapods, radius of curvature actuation

PI M-840
(sold with PC and Hexapod.py control code)
Physik Instrumente

6 actuators control 6 rigid body degrees of freedom
Custom secondary mirror control
Ultra-precise brain surgery
Physik Instrumente

ROC: rigid tripod fixed to back

PM Segments: Hexapod + ROC
SM: Hexapod only
Hexapods, radius of curvature actuation

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6 actuators control 6 rigid body degrees of freedom
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ROC: rigid tripod fixed to back
Push/pull between X and Y with actuator

PM Segments: Hexapod + ROC
SM: Hexapod only
NIRCam

Two wheels: “Pupil” & “Filter”
Weak Lenses to add defocus
Bandpass: few % $d\lambda/\lambda$ filter ~2μm
$K$ ~ 15 (TBD)
Barely OK co-phased PSF

Strehl Ratio: Peak intensity (actual aberration) / Peak intensity (perfect wavefront) (on the same pupil with secondary obstructions/gaps/transmission)
Perfectly co-phased PSF

Strehl Ratio: Peak intensity (actual aberration) / Peak intensity (perfect wavefront) 
(on the same pupil with secondary obstructions/gaps/transmission)
Typical WFS data

~1-10 minute CR splits without dithering

Median filter CRs

Dark-subtract

Subtract ‘0th read’ (DCS) or fit up-the-ramp

Flatten - fix bad pixels

WFSC EXEC @ STScI (JPL-written)

Limits on routine wavefront sensing with NIRCam on JWST
SPIE 5487-149 2004 (Glasgow)
WFS data to WFC commands

Visits being executed
By JWST Observing
Plan Executive (OPE)

WFSC EXEC

R. Makidon
WFSC WG
24 Feb 2005
Part II - PSF theory

**Optical Path Difference**
This is the deviation of the wavefront from 'perfect'... when talking of an image being formed by a converging wavefront,

THE DEVIATION OF THE WAVEFRONT FROM THE PERFECT SPHERICAL CONVERGING WAVE

is the optical path difference.

In a collimated beam such as an interferometer, the deviation of a wavefront from the perfect, flat wavefront is the OPD.

OPD(x,y) is a real function in 'pupil space', dimensions of LENGTH usually
At wavelength it is expressed in RADIANS of PHASE: \( \phi(x,y) = \left(\frac{2 \pi}{\lambda}\right) \text{OPD}(x,y) \)
PSF theory (cont’d)

Wave optics (scalar field, Fraunhofer approximation)

Monochromatic plane wave propagating in z direction \( \sim \exp \{ i (kz - \omega t + \phi(x,y)) \} \)
Aperture \( A(x,y) \): real function
Phase \( \phi(x,y) \): real function

Electric field over aperture: \( E(x,y) = A \exp \{ i \phi(x,y) \} \) (complex number, encodes phase lag with the ‘angle’ part of the complex number) - useful fiction
Intensity \( I \sim E E^* \) (real positive) - measurable

If phase is constant over aperture: perfect wavefront resulting in perfect PSF

Image field strength = \( a(k) = \text{FT}(E(x,y)) \). \( k \) is 2-d vector in angle space (radians)
John Krist calls \( a(k) \) the “Amplitude Spread Function”. \( \text{PSF}(k) = a a^* \) - real positive
Think Fourier
Sine wave aberration is a pair of delta functions in its ‘Fourier transform domain’
At small amplitudes this corresponds to pair of bright spots in the PSF: pupil: \(\exp(i\phi) \sim 1 + i\phi\) image: \(\delta(0) + \text{FT}(\text{sine})\)
As size of aberration increases, \(\exp(i\phi)\) expansion gets higher order terms. Quadratic terms produce spots at twice the separation...
What is a PSF?


Aperture $A(x,y)$: real function
Phase $\phi(x,y)$: real function

Electric field over aperture: $A \exp(i\phi)$

For $\phi < 1$ truncate expansion of $\exp(i\phi)$ at second order in $\phi$:

$$A_{AO} = AA_\phi = A(1 + i\phi - \phi^2/2 + ...).$$

FT this to get image plane electric field
What is a PSF?


A, a are FT pairs  \( \Phi, \phi \) are FT pairs  star is convolution

\[
\begin{align*}
p_{AO} &= p_0 + p_1 + p_2 \\
      &= aa^* \\
      &-i[a(a^* \ast \Phi^*) - a^*(a \ast \Phi)] \\
      + (a \ast \Phi)(a^* \ast \Phi^*) \\
      - \frac{1}{2}[a(a^* \ast \Phi^* \ast \Phi^*) + a^*(a \ast \Phi \ast \Phi)], \\
p_1 &= -i[a(a^* \ast \Phi^*) - a^*(a \ast \Phi)] = 2\text{Im}[(a(a^* \ast \Phi^*))], \\
p_2 &= (a \ast \Phi)(a^* \ast \Phi^*) - \frac{1}{2}[a(a^* \ast \Phi^* \ast \Phi^*) + a^*(a \ast \Phi \ast \Phi)].
\end{align*}
\]
What is a PSF?

\[ A_{AO} = AA_{\phi} = A(1 + i\phi - \phi^2/2 + ...). \]

\[ a_{\alpha \phi} = \sum_{k=0}^{\infty} \frac{i^k}{k!}(a \ast^k \Phi), \]

\[ p_{AO} = \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \frac{i^k}{k!}(a \ast^k \Phi) \frac{(-i)^j}{j!}(a^* \ast^j \Phi^*). \]

\[ p_{AO} = \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{i^k(-i)^{n-k}}{k!(n-k)!}(a \ast^k \Phi)(a^* \ast^{n-k} \Phi^*). \]

\[ p_{\alpha} = i^n \sum_{k=0}^{n} \frac{(-1)^{n-k}}{k!(n-k)!}(a \ast^k \Phi)(a^* \ast^{n-k} \Phi^*). \]


Python/Numarray/pyfits/matplotlib
So what?

Calculate the phase in the pupil plane from imaging data -

“Image-based phase retrieval”

Curvature Sensing
Roddier

Focus-diverse phase retrieval
“Phase diversity”
Paxman, Fienup, Gonsalves

Extra-curvature
Brighter
Early focus
Dimmer
Regular focus
Extra-focal images

A +2w
B -2w
OPD
rotsub(A,B)

image
image
pupil
image
Choosing the amount of defocus

Numerical experiment

Place a sinusoidal phase aberration over the pupil and try three different amounts of defocus.

pre-focus image - rotated post-focus image = signal
What is the best defocus to use?

Signal strength for given spatial frequency of aberration (number of ripples across mirror) is periodic in 1/defocus

B. Dean, C. Bowers, “Diversity Selection for Phase-Diverse-Phase-Retrieval,” JOSA, 20(8), 2003, pp. 1490-1504
PSF examples

Aberrations - linear stretch +/- 1 micron rms segment error (wavefront)

1
1 reduced tilts
1/2
140nm (RQ)
50nm (~perfect)

1
1 reduced tilts
end of DHS?
SR ~80%
SR ~97%

2 micron monochromatic PSFs (simple FFTs at Nyquist sampling)
Misell-Gerchberg-Saxton (MGS) algorithm

MGS in this case - a mapping from one guess at the phase, $f_1(x)$, to a better estimate, $f_2(x)$, using a known pupil function and image data $I(k)$

- Assume a pupil function $A(x)$ and a first-guess phase $f_1(x)$
- Calculate $a(k) = F[(A(x) \exp\{i f_1(x)\})] = b(k) \exp\{i g(k)\}$ \textit{(b is real)}
- Use measured data for intensity $I(k)$ - replace $b(k)$ with $\sqrt{I(k)}$
- Now we have $\sqrt{I(k)} \exp\{i g(k)\}$
- Back-transform it - we write this as $C(x) \exp\{i f_2(x)\}$
- This gives us our revised estimate of the phase, $f_2(x)$

- Now write the pupil field using known pupil $A$ instead of $C$: $A(x) \exp\{i f_2(x)\}$
- And do the same operations to get the next estimate, $f_3(x)$, for the phase

Keep going till you are happy. It WILL converge but not necessarily to the right phase -

- Incorrect $A(x)$
- Improperly reduced data $I(k)$ (CR, flats, photon noise, real pixel response,...)
- Difference between FFT samples and physical pixels, etc.

Gerchberg, R. H. & Saxton, H. O. 1972, Optik, 35(2), 237
Developing MGS intuition - I

SR 78% - below par by 2%

Crank up the defocus…

OPD from “Limits on routine wavefront sensing with NIRCam on JWST” A. Sivaramakrishnan, E. C. Morse, R. B. Makidon, L. E. Bergeron, S. Casertano, D. F. Figer, D. S. Acton, P. D. Atcheson, and M. J. Rieke SPIE 5487-149 2004 (Glasgow)
Developing MGS intuition - II

Gaussian bump on mirror
Same height bump, different widths of bump
Aperture is 192 pixels dia
Bump at half a radius out
Bump height 1 radian at 2um

+/− defocus amount in waves

Python/Numarray/pyfits/matplotlib
Rules of thumb, definitions

- Resolution element (Res Elt) $\lambda / D$ radians
  - $0.2 \lambda$ (in microns) / $D$ (in meters) in arcseconds
- Nyquist sampling
  - Astronomer’s version - 2 pixels across $1.22$ Res Elts
  - Nyquist’s version: 2 samples per Res Elt
- Effective focal length
  - $F = f D$ (where $f =$ focal ratio)
- Pixel size $p$
  - Angular size on sky = $p / F$ radians
  - $0.2 p$ (in microns) / ($f D$ (in meters))
- Strehl ratio
  - Peak intensity (actual aberration) / Peak intensity (perfect wavefront)
  - Marechal approximation: $SR \sim \exp - (\text{phase variance}) = 1 - \sigma^2$
- Diameter of a defocussed image:
  - $\sim 8$ (peak-to-valley defocus in waves) Res Elts
### Rough photometric zero points


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<th>l/um</th>
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<th>W m-2 um-1</th>
<th>Jy</th>
<th>photons s-1 m-2 um-1</th>
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