SISD Training Lectures in Spectroscopy

Anatomy of a Spectrum

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Visual Spectrum of the Sun

Jeff Valenti
Blue Spectrum of the Sun

Morphological Features in Spectra

Line Flux = \int_{\lambda_i}^{\lambda_f} F_\lambda d\lambda

(Units: erg s\(^{-1}\) cm\(^{-2}\))

Continuum Fit

Continuum

Absorption Lines

Emission Lines

Flux (10\(^{-1}\) erg s\(^{-1}\) cm\(^{-2}\) A\(^{-1}\))
Residual Intensity is the Flux Spectrum Divided by Continuum Fit

Line Width

Line Depth

Equivalent Width:

$$W_{eq} = \int (1 - R) \, d\lambda$$

(Wavelength [Å]

Wide Variety of Continuum Shapes
Planck Function

Assumptions
¥ Uniform temperature source
¥ Source is opaque

Mathematical description

\[ B_\lambda = \frac{2hc^2 / \lambda^5}{\exp(hc / \lambda kT) - 1} \]

(Units: \( \frac{\text{erg}}{\text{s} \text{cm}^2 \text{Å ster}} \))

\( h \) = Planck Constant = \( 6.63 \times 10^{-27} \) erg s
\( c \) = Speed of Light = \( 3.00 \times 10^{10} \) cm s\(^{-1}\)
\( k \) = Boltzmann Constant = \( 1.38 \times 10^{-16} \) erg K\(^{-1}\)
\( \lambda \) = Wavelength of Light (cm)
\( T \) = Uniform Temperature (K)

Computed Blackbody Spectra

\[ B_\lambda = \frac{2hc^2 / \lambda^5}{\exp(hc / \lambda kT) - 1} \]
Wien Displacement Law

- Blackbody peak wavelength inversely proportional to temperature
- Find peak wavelength by solving:

\[
\frac{dB_\lambda}{d\lambda} = 0 \quad \text{where} \quad B_\lambda = \frac{2hc^2 / \lambda^5}{\exp(hc / \lambda kT) - 1}
\]

\[5(1 - e^{-y}) = y \quad \text{where} \quad y = \frac{hc}{\lambda_{pk} kT}
\]

Numerical solution: \( y = 4.97 \)

Wien Law:
\[\lambda_{pk} T = 0.29 \text{ cm K} \]

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>Peak Wavelength (\mu m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>3000</td>
<td>1</td>
</tr>
<tr>
<td>30,000</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Stellar Flux Spectra vs. Planck Function
Spectral Features due to Hydrogen

Laboratory Spectra to Identify Lines
Line Identification References

« Labelled Spectral Atlases
¥ Solar Photosphere, 0.36-22 micron (Wallace et al. 1991-1999)
¥ Sunspot, 0.39-21 micron (Wallace et al. 1992-2000)
¥ Arcturus (K1 III), 0.37-5.3 micron (Hinkle et al. 1995-2000)
« Printed Line Lists
¥ Atomic and Ionic Spectrum Lines Below 2000 A (Kelly 1987)
¥ FUV lines in solar spectrum (Sandlin et al. 1986, ApJS, 61, 801)
¥ Ultraviolet Multiplet Table (Moore 1950)
¥ A Multiplet Table of Astrophysical Interest (Moore 1945)
« Electronic Media
  http://www.astro.univie.ac.at/~vald/
  http://www.hia.nrc.ca/staff/dcm/atomic_data.html

Definition of Spectral Resolution

Resolution:
\[ R = \frac{\Delta \lambda}{\lambda} \quad \text{or} \quad \frac{c \Delta \lambda}{\lambda} \]
(Units: Å or km s\(^{-1}\))

Resolving Power:
\[ R = \frac{\lambda}{\Delta \lambda} \]
(Units: Dimensionless)
Line Spread Function (LSF, IP, PSF)

- Intrinsic Profile
- Observed Profile
- Line Spread Function (LSF)
- Convolution Sum of Shifted and Scaled LSFs
- Nyquist Sampling

Resolution vs. Sampling
Coverage vs. Redundancy

Dispersion \( \frac{\lambda}{2R} \)

Effect of Resolution and Sampling

- \( R = 100,000 \) - 0.0025 \( \AA \) = 3k l/mm
- \( R = 10,000 \) - 0.025 \( \AA \) = 30 l/mm
- \( R = 1000 \) - 0.25 \( \AA \) = 300 l/mm
- \( R = 100 \) - 2.5 \( \AA \) = 3000 l/mm

Resolution vs. Sampling
Coverage vs. Redundancy

Dispersion \( \frac{\lambda}{2R} \)
**Opacity**

- **Absorption coefficient**: \( \alpha \)
  - Fractional decrease in intensity per unit distance travelled
  \[
  \alpha = \frac{-\Delta I}{\Delta x} \quad \leftrightarrow \quad \Delta I = -\alpha I \Delta x
  \]

\[\alpha = n \sigma \quad \text{(Units: cm}^{-1}\text{)} \]

- \( n \): Number density of Absorbers
- \( \sigma \): Cross-sectional Area of Absorber
- Units: \( \text{cm}^{-1} \)

Note: \( \sigma, \alpha, \) and \( I \) can be functions of \( \lambda \) and \( x \).

**Mean Free Path**

- Approximate path length required to absorb beam.

\[
l = \frac{1}{\alpha} = \frac{1}{n \sigma}
\]

Electrons locked up
In neutral molecules

- \( \sigma \ll \sigma_T \)

- **Example:**
  - For electrons: \( \sigma_T = 7 \times 10^{-25} \text{ cm}^2 \)
  - In this room: \( n_{\text{Room}} = 3 \times 10^{10} \text{ cm}^{-3} \)
  - Absorption coefficient: \( \alpha = n_{\text{Room}} \sigma_T = 2 \times 10^{-5} \text{ cm}^{-1} \)
  - Mean free path: \( l = \frac{1}{\alpha} = 5 \times 10^4 \text{ cm} = 0.5 \text{ km} \)
Definition of Optical Depth

Recall: \[ \Delta I = -\alpha \Delta x \]

As \( \Delta x \to 0 \) this becomes
\[ \frac{dI}{I} = -\alpha dx \]

Solving for \( I \) gives:
\[ \frac{I(x)}{I(0)} = e^{-\tau} \]

where \( \tau(x) = \int_0^x \alpha dx \) is optical depth.

**Optical depth is the number of times light from a source has been diminished by a factor of \( 1/e = 37\% \).**

Optical depth is **dimensionless**.

Optical Depth Examples

**Examples:**

At \( \tau = 0.1 \) intensity has dropped to \( e^{-0.1} = 90\% \) of \( I_0 \)
At \( \tau = \frac{2}{3} \) intensity has dropped to \( e^{-0.7} = 50\% \) of \( I_0 \)
At \( \tau = 3.0 \) intensity has dropped to \( e^{-3.0} = 5\% \) of \( I_0 \)

**Optically thin** means minimal absorption: \( \tau \ll 1 \)

**Optically thick** means complete absorption: \( \tau \gg 1 \)

How far do we see into a star?
To the **Depth of Formation**, where \( \tau = \frac{2}{3} \)

Depth of formation is a strong function of wavelength.
Structure of the Solar Atmosphere

Behavior or spectrum driven by wavelength dependence of depth of formation

Spectral Features due to Hydrogen

Hydrogen Atom

Wavelength (Å)

Visible

NUV

FUV

Fe I

C IV

Temperature Minimum

Mass Column (g cm⁻²)

Temperature (K)

Depth Of Formation Of Continua
Analysis of Vega Spectrum

« Strong Balmer series and Balmer jump (transitions from $N = 2$)
¥ Seeing much higher, cooler, fainter layers in lines
¥ Balmer opacity is large in A-type stars. Why?

« Recall: $\alpha = n\sigma$
¥ $\sigma$ is an atomic property that is identical for all stars
¥ $n$ is actually the product of several factors:

$$n = \text{total number density of particles}$$
$\times$ abundance (hydrogen is 90% by number)
$\times$ neutral fraction (~50%, ~100% for Sun)
$\times$ excitation (fraction of hydrogen in $N = 2$)

¥ Excitation of $N = 2$ must be high. Why?

Boltzmann Factor (Excitation)

« Relative population in level $N$: $p_N = \sum N \exp\left(-\frac{E_N}{kT}\right)$

« Excitation fraction: $f_N = \frac{p_N}{p_1 + p_2 + p_3 + \ldots} = \frac{p_N}{\text{Partition Function}}$

« For hydrogen: $E_1 = 0$, $E_2 = 10.2$ eV, $E_3 = 12.1$ eV, ...

$U = 2$, since $E_N \ll kT$ 

$$f_N \approx N^2 \exp\left(-\frac{E_N}{kT}\right)$$

$$f_e \approx 4 \exp\left(-\frac{118,000}{T}\right)$$

¥ Sun: $T = 5770$ K $\rightarrow f_e = 4e^{-20.5} = 6 \times 10^{-9}$
¥ Vega: $T = 10,000$ K $\rightarrow f_e = 4e^{-11.8} = 3 \times 10^{-5}$

« Vega has 5000 times as much hydrogen in $N = 2$.
« Additional heating increases excitation, but neutral fraction drops.
Radiative Transfer

- Recall: \[ \frac{dI}{I} = -\alpha \, dx = -d\tau \]
- Transfer Equation:
  \[ \frac{dI}{d\tau} = -I + S \]

- If collisions are more frequent than photon emission:
  - System is in Local Thermodynamic Equilibrium (LTE)
  - Calculate \( n(x) \) from \( T(x) \)
  - Calculate \( \tau(x) \) from \( n(x) \) and \( \alpha \)
  - Local emission (source function) is Planck function: \( S = B_\lambda \)
  - Solve transfer equation for \( I(x) \), especially at the surface!

- Otherwise system has non-LTE (NLTE) characteristics.
  - \( I, S, \tau, n, \) and \( T \) are interrelated — very messy!

Stellar Parameters

- Stellar parameters that affect synthetic spectrum
  - Effective temperature (via ionization and excitation)
  - Gravity (high gravity gives broad line wings due to collisions)
  - Abundances or a global metallicity (affects opacity in lines)
  - Magnetic fields (changes wavelength dependence of opacity)
  - Microturbulence and Macroturbulence (Doppler smearing)
  - Rotation (more Doppler smearing)
  - Radial velocity (Doppler shift)

- Spectroscopy Made Easy (SME)
  - Fit observations or just synthesize a spectrum
  - Atomic data still a pain
  - Valenti & Piskunov (1996), A&AS, 118, 595
Effect of Rotation

![Graph of Effect of Rotation](image)