Improving solar physics by studying other stars

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IAU-GA-SP13 – Beijing – August 2012

Selected (two) topical problems in solar/stellar modelling

- The effect of the surface layers on the oscillation frequencies
- Asteroseismic ages and heavy-element abundances of the Sun & solar-like stars

Observed solar frequencies

m-averaged frequencies from MDI instrument on SOHO

1000 \( \sigma \) error bars

\[ n = 0 \] (f mode)
\[ n = 1 \]

spherical degree

in distant stars typically only low-degree (\( l = 0,1,2,3 \)) modes available

Observed solar frequencies

Courtesy of J. Christensen-Dalsgaard (C-D)
Observed solar frequencies (power spectrum)

in distant stars typically only low-degree ($l=0,1,2,3$) modes available

VIRGO on SOHO (whole-disk):

Examples of Kepler solar-like pulsators

(3-month time series)

16 Cyg A

16 Cyg B

Metcalfe et al. (2012)

Solar-like pulsators: surface effects

Solar observations – adiabatic calculations

Christensen-Dalsgaard et al. (1996)

Solar-like pulsators: surface effects

C-D & Gough (1980): surface contribution $\delta \nu$ from a plane-parallel polytropic layer supporting an isothermal atmosphere:

$$\delta \nu \equiv \nu_0 [\Gamma(m+1)\Gamma(m+2)]^{-1} \left( \frac{\mu}{2 \nu_0} \right)^{2(m+1)}$$

Kjeldsen et al. (2008):

$$\delta \nu(n) \equiv \nu_{0,\text{obs}}(n) - \nu_{0,\text{mod}}(n) \approx \nu \left[ \frac{\nu_{0,\text{obs}}(n)}{\nu_0} \right]$$
Solar-like pulsators: surface effects

C-D & Gough (1980): surface contribution $\delta \nu$ from a plane-parallel polytropic layer supporting an isothermal atmosphere:

$$\delta \nu = \nu_0 \Gamma(m+1) \Gamma(m+2) \left( \frac{1}{2} \frac{\nu_0}{\nu_{cp}} \right)^{2(m+1)} \approx \beta \left( \frac{\nu_0}{\nu_{cp}} \right)^{b}$$

Kjeldsen et al. (2008):

$$\delta \nu = \nu_0 - \nu_{cp} \Rightarrow \frac{\nu_{obs}(n)}{\nu_0} \approx \beta \left( \frac{\nu_0}{\nu_{cp}} \right)^{b}$$

Observations – solar model

BiSON data: $b = 4.82$
- radial modes
- dipole modes

Solar-like Kepler pulsators: surface effects

Empirical power law: $\delta \nu = \nu_{obs} - \nu_{ref} \approx a \left( \frac{\nu_{obs}}{\nu_0} \right)^{b}$

KIC 3632418

Mathur et al. (2012)

Empirical power law: $\delta \nu = \nu_{obs} - \nu_{ref} \approx a \left( \frac{\nu_{obs}}{\nu_0} \right)^{b}$

KIC 6106415

Mathur et al. (2012)

Empirical power law: $\delta \nu = \nu_{obs} - \nu_{ref} \approx a \left( \frac{\nu_{obs}}{\nu_0} \right)^{b}$
Solar-like pulsators: surface effects

Momentum equation of stellar (envelope) structure:
\[
\frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla P + \mathbf{f}
\]
(mean) turbulent momentum flux (turbulent pressure):
\[
\rho \mathbf{u} = \rho \mathbf{u}^{\text{turb}}
\]

Nonadiabaticity:
\[
\frac{\partial}{\partial t} \left( \frac{\delta L}{L} \right) - \frac{\delta T}{T} \frac{\delta L}{L} - \mathbf{v} \cdot \frac{\partial}{\partial x} \delta \mathbf{v} = \frac{\delta T}{T} \left( \frac{\delta p}{p} \right) + \frac{\delta \rho}{\rho}
\]

Convection dynamics:
\[
\frac{\partial}{\partial t} \left( \frac{\delta L}{L} \right) - \frac{\delta T}{T} \frac{\delta L}{L} - \mathbf{v} \cdot \frac{\partial}{\partial x} \delta \mathbf{v} = \frac{\delta T}{T} \left( \frac{\delta p}{p} \right) + \frac{\delta \rho}{\rho}
\]
Solar-like pulsators: surface effects

Nonadiabaticity:

\[
\frac{\partial}{\partial m} \left( \frac{\delta (L_0 + L_0)}{L} \right) = -\frac{\Delta T}{T} \delta \ln \rho + \frac{\delta \ln \rho}{\rho} \]

Convection dynamics:

\[
\frac{\partial}{\partial m} \left( \frac{\delta L_0}{L} \right) = -\frac{\Delta T}{T} \delta \ln \rho + \frac{\delta \ln \rho}{\rho} \]

Asteroseismic solar/stellar ages

\[
\Delta \nu \, \delta \nu \text{ diagram}
\]

Christensen-Dalsgaard (1994)

\[
\alpha \text{Cen} A \quad \alpha \text{Cen} B \quad \text{Sun}
\]

(i) \( p^i \) contribution in the overshoot region

(ii) opacity effect: "back warming"
Asteroseismic ($\Delta \nu, \delta \nu$) diagram

Monteiro et al. (2002)

Age-sensitive diagnostics of the stellar structure

- asymptotic p-mode frequency behaviour ($n/L \to \infty$): $|L^2 - l(l+1)|$

$$\nu \simeq (n + \frac{1}{2} + \epsilon) \Delta \nu - \frac{A L^2 - B}{\nu} (\Delta \nu)^3$$

$$A \propto \frac{1}{\Delta \nu} \int_0^R \frac{1}{r} \frac{dc}{dr} dr$$

Modelling global stellar parameters ($R, M, \text{age}$)

Non-seismic observational constraints (input from spectroscopy): $T_{\text{eff}}, \log g, [\text{Fe/H}]$

(input from parallaxes & galactic extinction): $L$

<table>
<thead>
<tr>
<th>Method</th>
<th>seismic input</th>
<th>median (statistical) uncertainties (%)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$\nu_{\text{max}}$</td>
<td>$\Delta \nu$</td>
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<tr>
<td>Grid modelling:</td>
<td></td>
<td></td>
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<tr>
<td>(a) RADIUS (Stello et al. 2009)</td>
<td>✓</td>
<td></td>
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<tr>
<td>(b) Yale-Birmingham (Gai et al. 2011)</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>(c) SEEK (Quirion et al. 2010)</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Fitting all observed frequencies:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AMP (Asteroseismic Modeling Portal)</td>
<td>✓</td>
<td>✓</td>
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</tbody>
</table>

Mathur et al. 2012

Age-sensitive diagnostics of the stellar structure

- asymptotic p-mode frequency behaviour ($n/L \to \infty$): $|L^2 - l(l+1)|$

$$\nu \simeq (n + \frac{1}{2} + \epsilon) \Delta \nu - \frac{A L^2 - B}{\nu} (\Delta \nu)^3$$

$$A \propto \frac{1}{\Delta \nu} \int_0^R \frac{1}{r} \frac{dc}{dr} dr$$
Age-sensitive diagnostics of the stellar structure

- asymptotic p-mode frequency behaviour \( (n/L \to \infty) \int \nu^2 = l(l+1) \)

\[ \nu \simeq (n + \frac{1}{2} \varepsilon + \epsilon^2) \Delta \nu - A \frac{L^2 - B}{\nu} \Delta \nu \]

- evolutionary computations depend on 3 initial parameters: e.g., \( Y_0, Z_0 \) and \( \alpha_c \)
- Calibrated (L, R) models: \( Z_0(Y_0, \alpha_c) \) at any \( t_\star \) 2-parameter set of models \((Z_0, t_\star)\)

4.15 Gy
4.60 Gy
5.10 Gy

at constant \( Z_0 \)

Gough & Novotny (1990)

Houdek & Gough (2011)

Houdek & Gough (2007)
Age-sensitive diagnostics of the stellar structure

- **Calibration** using combinations of the seismically determined parameters

\[
\zeta_\alpha = (A, C, F, \frac{\gamma}{\chi}), \quad \alpha = 1, 2, 3, 4
\]

\[
\gamma = \log(p) / \log(\rho)
\]

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Seismic diagnostic

Adiabatic exponent \( \gamma_1 \)

\[
\gamma_1 = (\partial \ln p / \partial \ln \rho)_s
\]

\( \delta \gamma_1 \) produces oscillatory glitch contribution

\[
\delta \gamma_1 = \gamma_{\odot} - \gamma_0
\]

Seismic diagnostic models:

- \( \delta \gamma_1 \)
- \( \delta \gamma_1 \)

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Applying the seismic diagnostic to low-degree \( p \) modes: Sun

Seismic diagnostic (GH & Gough 2007)

\[
\delta \nu = \delta \nu_1 + \delta \nu_{\text{He}} + \delta \nu_{\text{HeII}} + \delta \nu_{\text{HeII}}
\]

\[
\Delta \nu = \delta \nu_{\text{HeII}} - 2 \delta \nu_{\text{He}} + \delta \nu_{\text{HeII}}
\]

\[
\frac{\delta \gamma_1}{\gamma_1} \propto Y \quad \text{helium abundance}
\]

For BiSON data:

- \( \delta \gamma_1 / \gamma_1 |_{\text{HeII}} = 0.045 \)
- \( \gamma_{\text{HeII}} = 819 \pm 6 \)
- \( \gamma_{\text{He}} = 2310 \pm 8 \)

For Model S:

- \( \delta \gamma_1 / \gamma_1 |_{\text{HeII}} = 0.017 \)
- \( \gamma_{\text{HeII}} = 816 \pm 8 \)
- \( \gamma_{\text{He}} = 2287 \pm 8 \)
Solar/stellar age calibration

- approximate solar value $\xi_0$ by a two-term expansion about reference value $\xi_0^*$. 

$$\xi = \xi_0 + \frac{\partial \xi_0}{\partial t} \Delta t_0 + \frac{\partial \xi_0}{\partial Z} \Delta Z_0 = \xi_0^*.$$ 

- and the solution is:

$$t_0 = t_{ref} + \Delta t,$$

$$Z_0 = Z_{ref} + \Delta Z,$$

from reference model.

Results after five iterations using BiSON data & $(A,C,F_{\alpha}$)

<table>
<thead>
<tr>
<th>$t_0$ (Gy)</th>
<th>$Z_0$</th>
<th>$Y_0$</th>
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<tbody>
<tr>
<td>6.01 ± 0.04</td>
<td>0.25 ± 0.04</td>
<td>0.26 ± 0.04</td>
</tr>
</tbody>
</table>

Cheikh et al. (2009) | 0.61 ± 0.09 | 0.62 ± 0.09 |

Korzul & Goupil (2011) | 0.61 ± 0.09 | 0.62 ± 0.09 |

Asplund et al. (2009) | 0.61 ± 0.09 | 0.62 ± 0.09 |
**Age calibration Summary**

- **Simulated SONG data**
- GH & Gough (2009)
- Improvement by $\geq 2$ of age accuracy

- Including the seismic signatures of the two stages of helium ionization substantially improve the calibration of stellar ages and abundances.

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**Summary/Conclusions**

- Observed surface effects in Kepler stars can contribute to improve our understanding of the near-surface physics in the Sun & solar-type stars.

- Mean turbulent pressure dominating surface effect; nonadiabaticity and convection dynamics must also not be neglected.

- Including the seismic signatures of the two stages of helium ionization substantially improve the calibration of stellar ages and abundances.

- This seismic calibration procedure can be applied to data from CoRoT, Kepler and planned observing campaigns (SONG).