Black Hole Formation and Growth: Simulations in General Relativity

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Black Holes
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Introduction and Motivation

- BHs are found everywhere:
  - compact binary X-ray sources;
  - GRBs;
  - quasar & AGN central engines;
  - bulge galaxy cores;
  - BHBH and BHNS binaries;
  - Large Hadron Collider??

- **Strategy:**
  BHs are strong-field objects governed by Einstein’s theory of general relativity.
  $\Rightarrow$ GR simulations of
  - collapse to BHs,
  - BH binary merger and recoil,
  - BH accretion, etc.,
  may help reveal how, when and where BHs
  $\rightarrow$ form and grow;
  $\rightarrow$ can be detected.

- **Two examples:**
  - collapse of hypermassive NSs to BHs;
  - formation & growth of SMBH seeds.
Differentially Rotating ‘Hypermassive’ Stars


Example: \( \Gamma = 2 \) polytrope; \( \Omega / \Omega_c \approx \frac{1}{1 + (\omega / \lambda)^2} \)

Conclude: ‘hypermassive’ stars can significantly exceed the spherical TOV mass limit.

Questions: are they stable? how do they form?
$3+1$ (ADM) Field Eqns

\[ ds^2 = -\alpha^2 \, dt^2 + \gamma_{ij} (dx^i + \beta^i \, dt)(dx^j + \beta^j \, dt) . \]

- **Constraint Equations**
  \[ R + K^2 - K_{ij} K^{ij} = 16\pi \rho \quad \text{(Hamiltonian)}, \]
  \[ D_j (K^{ij} - \gamma^{ij} K) = 8\pi S^i \quad \text{(Momentum)} . \]

- **Evolution Equations**
  \[ \partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i , \]
  \[ \partial_t K_{ij} = \alpha R_{ij} + \cdots - 8\pi \alpha [S_{ij} - \frac{1}{2} \gamma_{ij}(S - \rho)] . \]

- **Gauge Quantities**
  \[ \alpha, \quad \beta^i \]
Modifying ADM Field Eqns

Shibata & Nakamura 1995; Baumgarte & Shapiro 1999

(BSSN)

- Conformal Decomposition: “York-Lichnerowicz split”

\[ \tilde{\gamma}_{ij} = e^{-4\phi}\gamma_{ij}, \quad \text{where} \quad e^{4\phi} = \gamma^{1/3}, \]

\[ \tilde{A}_{ij} = \tilde{K}_{ij} - \frac{1}{3}\tilde{\gamma}_{ij}K \]

- Connection Functions

\[ \tilde{\Gamma}^i \equiv \tilde{\gamma}^{jk}\tilde{\Gamma}^i_{jk} = -\partial_j\tilde{\gamma}^{ij}, \]

- Evolve

\[ \tilde{\gamma}_{ij}, \quad \tilde{A}_{ij}, \quad \phi, \quad K, \quad \& \quad \tilde{\Gamma}^i \]

- Advantage

\[ \tilde{R}_{ij} = -\frac{1}{2}\tilde{\gamma}^{lm}\partial_m\partial_l\tilde{\gamma}_{ij} + \tilde{\gamma}_k(\partial_i\partial_j)\tilde{\Gamma}^k + \cdots, \]

\[ \Rightarrow \partial_t^2\tilde{\gamma}_{ij} \sim \partial_t\tilde{A}_{ij} \sim \tilde{R}_{ij} \sim \nabla^2\tilde{\gamma}_{ij} \]

- Result: dramatically improved stability
**Conclusions:**

- At least some hypermassive stars are dynamically stable.
- **BUT** they are all secularly unstable to $J$-redistribution via turbulent viscosity, magnetic braking &/or GWs;
  - → delayed collapse to BHs?
  - → delayed GW burst?
NS-NS Merger: Formation of a Hypermassive NS

Faber & Rasio (2000): PN; Faber et al. 2004: CFGRT
Shibata* et al. 2003; 2005; 2006: GRT

*Simulations: $0.65 \leq M_1/M_2 \leq 1$, realistic EOS

- $M < M_{\text{thr}} \approx 1.3M_{\text{max}}^{OV} \approx 2.8M_\odot \Rightarrow$ HMNS;
- $\Gamma > 2.24 \rightarrow$ triaxial ellipsoid $\rightarrow$ GWs.
- shown above: $M_{ADM} = 2.7M_\odot$, $P = 2.11$ ms;
Collapse of a Magnetized Hypermassive Star

Duez, Liu, Shapiro, Shibata & Stephens (2006a,b): axisymmetry

- **Initial Seed B Field**
  - **Topology:** purely poloidal
  - **Strength:** \( C \equiv \max \left[ \frac{B^2}{4\pi P} \right] = 2.5 \times 10^{-3} \)

- **B-field Amplification:**
  - **Winding:** \( \tau_A = \frac{R}{v_A} \)
  - **MRI:** \( \tau_{\text{MRI}} \sim P_c \ll \tau_A \) (Balbus & Hawley 1991)

- **Computational Challenge**
  - **Wavelength:** \( \lambda_{\text{MRI}} = \frac{2\pi v_A}{\Omega} \sim \frac{R}{10} \)
  - **Resolution Requirement:** \( \Delta \lesssim \frac{\lambda_{\text{MRI}}}{10} \)

\[ \Rightarrow \] To follow collapse, the evolution time must exceed \( t_A \approx 75P_c \approx 3000M \).

\[ \Rightarrow \] To resolve the fastest growing MRI mode, we require \( N^2 \) zones with \( N \gtrsim 400 \).
NSNS Mergers & HMNSs: Plausible Routes For Short-Hard GRBs?

(figures adapted from Shibata & Taniguchi 2006)
**BHs as Central Engines For GRBs?**

Duez, Liu, Shapiro, Shibata & Stephens 2006a,b,c

- **GRBS: 2 classes** (BATSE, Swift, HETE, Chandra, HST)
  - **Long-Soft GRBs:**
    - \( \tau \sim 2 - 1000 \) sec;
    - in star-forming regions (spirals);
    - associated with SNs;
    - massive star collapse: ‘collapsars’ ?
    - Pop III collapse analogs?
  - **Short-Hard GRBs:**
    - \( \tau \sim 10 \) ms – 2 sec;
    - in low star-form. regions (ellipticals);
    - SN associations excluded;
    - NS-NSs → HMNSs? BH-NSs?

- **Exciting implications for Advanced LIGO!**
  - **Coincidence Detections:**
    - GW bursts + GRBs;
    - binary mergers: \( \sim 20 - 30 \) yr\(^{-1}\).

- **Simulations in full GRMHD:**
  - required & underway!
Rotating, Magnetized Stellar Collapse: Massive Pop III Star? Collapsar?

Liu & Shapiro 2007

**Initial Model** $M$:
- Max uniformly rot, marginally unstable $n = 3$ polytrope, $\langle B^2/4\pi P \rangle = 10^{-3}$.

**Final Model** ($t_{ex} = 29150M; t_f-t_{ex} = 1127M$):
- $M_h/M \approx 0.9, a_h/M_h \approx 0.7, M_d/M \approx 0.1$
- hot, massive disk + rotating BH + collimated $B$-field
→ **Long-Soft GRB candidate?**
**Origin of SMBHs: Clues and Constraints**

- **1\textsuperscript{st} SMBHs:**
  Existence of QSO SDSS 1148+5251 at $z_{QSO} = 6.43$ (Fan et al. 2003) \Rightarrow 1\textsuperscript{st} SMBHs formed by $t = 0.87$ Gyr in $\Lambda$CDM model.

- **Broad-line quasars with $0.1 \leq z \leq 2.1$:**
  SDSS sample of 12,698 quasars obeys the Edd limit, $L_{bol} \lesssim L_E$. (McLure & Dunlop 2004)

- **Radiation efficiency:**
  The luminosity density of quasars is $\sim 10\%$ the local SMBH mass density. (Soltan 1982; Yu & Tremaine 2002; Elvis et al. 2002)

\Rightarrow An appreciable fraction of the mass of a SMBH is likely acquired by (baryonic) disk accretion.

\Rightarrow The more massive the initial seed, the less time is required for it to grow to SMBH size by $z_{QSO} \geq 6.43$. 
One Possibility: a SMS, $M \gtrsim 10^4 M_\odot$.
Form when contracting gas builds up sufficient rad’n pressure to inhibit fragmentation & prevent star formation.
(e.g., Gnedin 2001; Bromm & Loeb 2003)

GR rotating collapse simulations:
max rotation yields a SMBH + disk,
$M_h/M \approx 0.9$, $a_h/M_h \approx 0.75$, $M_d/M \approx 0.1$.
(Shibata & Shapiro 2002)

Problems:
- SMSs have never been observed.
- Simulations $\Rightarrow$ 1$^{\text{st}}$ generation stars are Pop III stars, $M \approx 10^2 – 10^3 M_\odot$, not SMSs.
(Bromm et al. 2002; Abel et al. 2002; Yoshida et al. 2006)

Most conservative hypothesis:
Pop III stars $\rightarrow$ BH seeds (Madau & Rees 2001):
$M \sim 60 – 140$, & $\gtrsim 240 M_\odot$ (Heger et al. 2003);
$M \lesssim 600 M_\odot$ (Onukai & Palla 2003; Yoshida et al. 2003)
SMBH Spin Evolution

- **Significance:**
  efficiency of accretion & rate of SMBH growth depend sensitively on $a/M$.

- **Initial Conditions: Pop III stellar collapse**
  GR simulations $\Rightarrow 0 \leq a/M \lesssim 0.8$
  (Shibata & Shapiro 2002; Shibata et al. 2006)

- **Spin-up by major mergers**
  expect $M$ & $a/M \approx$ BHBH ISCO values
  (ISCO: Damour, Cook, Baumgarte, Grandclement, ...)
  $\Rightarrow a/M \approx 0.8 - 0.9$ for $M_1 = M_2$
  (simulations confirm: Pretorius, Campanelli, Centrella,...).

- **Spin-down by minor mergers**
  BH merging with many smaller BHs, isotropically distributed, $\Rightarrow a/M \sim M^{-7/3}$.
  (Hughes & Blandford 2003; Gammie et al. 2004)

- **Spin-equilibrium via accretion**
  $a/M = 1.0$, standard thin disk (Bardeen 1970);
  $a/M = 0.998$, + photon recap. (Thorne 1974);
  $a/M \approx 0.95$, turbulent MHD disk (De Villiers et al. 2004; Gammie, Shapiro & McKinney 2004).
GRMHD Flow Snapshot for $\alpha/M = 0.75$

McKinney & Gammie (2004); Gammie, Shapiro & Mckinney 2004
**SMBH Growth By Accretion**

- **Efficiencies:**
  \[
  \epsilon_M \equiv \frac{L}{\dot{M}_0c^2} = \epsilon_M(a/M), \quad \epsilon_L \equiv \frac{L}{L_E},
  \]

  \[
  \frac{dM}{dt} = (1 - \epsilon_M)\frac{d\dot{M}_0}{dt} = \left[\frac{\epsilon_L(1-\epsilon_M)}{\epsilon_M}\right] \frac{M}{\tau},
  \]

  where

  \[
  L_E = \frac{4\pi M\mu_e m_p c}{\sigma_T} \approx 1.3 \times 10^{46} \mu_e M_8 \text{ erg s}^{-1},
  \]

  \[
  \tau = \frac{M c^2}{L_E} \approx 0.45 \mu_e^{-1} \text{ Gyr}.
  \]

- **Mass Amplification at spin-equilibrium:**
  \[
  \frac{M(t)}{M(t_i)} = \exp \left[\frac{\epsilon_L(1-\epsilon_M)(t-t_i)}{\epsilon_M \tau}\right]
  \]

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Accretion-Driven Mass Amplification

$\Lambda$CDM

Accretion: $\epsilon_L = L/L_E = 1$; curve labels: $\epsilon_M = L/\dot{M}_0c^2 = \epsilon_M(a/M)$,

$M_i/M_\odot = 100 - 600$, $M_f/M_\odot = 10^9$;
dashed = pure accretion;
dotted = $10^4$ merger amplification $\times$ accretion.
SMBHs: Summary & Conclusions

- **Key issues:**
  - cosmological origin of seed SMBHs?
  - mass & spin evolution?
  - role in structure formation?

- **Clues & constraints:**
  - QSO 1148+5251: $z = 6.43$, $t = 0.87$ Gyr
  - $U_{QSO} \approx 0.1 \rho_{BH} c^2$
  - $M_{BH} - \sigma_*$ correlation
  - $M_{BH} - M_{bulge}$ correlation
  - etc.

- **Numerical GR:**
  mature enough (at last!) to probe physics underlying cosmological formation & growth of SMBHs, e.g.,
  - collapse to seed BHs;
  - BH binary merger and recoil;
  - gravitational wave generation;
  - BH accretion;
  - etc;