Figures of Merit
for Dark Energy Measurements

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What next for Dark Energy?

Theory
- Model Building
- Which flavor of DE?

Experiment
- Systematics control
- Experim. strategies

Phenomenology
- Parametrizations
- Statistical methods

Cosmo Probes
- SNe Ia, Weak Lensing
- CMB, BAO, clusters
# What next for Dark Energy?

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Dark Energy constraints: current status

Supernova Cosmology Project

**Union 08**
SN Ia compilation

**BAO**

**CMB**

**SNe**

Kowalski et al., arXiv:0804.4142
Dark Energy constraints: current status

Dataset (I)

Dataset (II)

redshift z

Zhao, Huterer & Zhang, arXiv:0712.2277
Figure of Merit:
definition, history, current status
FoM: requirements

- FoM should show intrinsic power of any given cosmological probe OR individual experiment to measure the properties of DE
- FoM should be as robust as possible w.r.t. fiducial DE model
- Should try to intuitively capture quantities/concepts we like to measure (e.g. variation in time of w)
- It should clearly differentiate between experiments/probes in a way that agrees with overall assessment
- It should, ideally, be represented by one number!

In sum, finding a suitable FoM is neither easy nor is there a unique choice
Early work

“If we are using SNe Ia alone to determine the cosmological parameters, then we clearly want to minimize the area of the error ellipse. “

volume of the ellipsoid : $V \propto \det(F)^{-1/2}$ where $F \equiv \left\langle -\frac{\partial^2 L}{\partial p_i \partial p_j} \right\rangle$

(Also showed how to get such a minimal-area/volume ellipse: for N cosmological parameters, need SNe located at N discrete, specific locations in z)

However, clearly there are other possibilities, e.g.:

“SN measurements will also be combined with other methods to determine cosmological parameters. A good example of the symbiosis is combining CMB measurements with those of SNe.” -> thinnest ellipse

Huterer & Turner, PRD, 2001
Smallest ellipse, thinnest ellipse

Huterer & Turner, 2001
Currently accepted FoM:
inverse area of ellipse in \( w_0 - w_a \) plane

The DETF figure of merit is defined as the reciprocal of the area of the error ellipse in the \( w_0 - w_a \) plane that encloses the 95% C.L. contour. (We show in the Technical Appendix that the area enclosed in the \( w_0 - w_a \) plane is the same as the area enclosed in the \( w_p - w_a \) plane.)

\[
 w(z) = w_0 + w_a (1 - a) = w_p + w_a (a_p - a) \]

\[
 \text{FoM} \equiv \frac{1}{\sigma(w_p) \times \sigma(w_a)}
\]

DETF report; Albrecht et al 2006
DETF FoM - advantages and disadvantages

The currently accepted FoM is a very reasonable choice which captures essential ingredients and is easy to compute.

However, we should also be aware of its deficiencies:

• DETF FoM probably fails to capture success at measuring models with non-canonical variations in \( w(z) \) at late times

• It definitely fails to capture success at measuring early DE

• It does not address anything about modified gravity vs. DE

• It doesn’t account for clustering of DE

• It’s not designed to measure deviations from LCDM
Other proposals/options for the dark energy FoM
Principal Components of \( w(z) \)

These are best-to-worst measured linear combinations of \( w(z) \)

Uncorrelated by construction

- Shows where sensitivity of any given survey is greatest
- Used by various authors to study optimization of surveys
- Used to make model-(in)dependent statements about DE

Huterer & Starkman 2003
Principal Components of $w(z)$

- $w(z)$
- Principal component
- $\text{error} / \text{(fiducial value)}$
- $\text{SNe + CMB + WL}$
- $\text{SNe + CMB}$
- $\text{SNe + WL}$
- $\text{WL + CMB}$
- $\text{SNe}$

(WL = Weak Lensing)
Uncorrelated measurements of Dark Energy evolution

Using Riess et al 2004 data

Cosmological constant case

Huterer & Cooray 2005
Other proposals for the FoM: using uncorrelated bandpowers

\[ \text{FoM} = \prod_{i=1}^{N_{\text{bins}}} \frac{1}{\sigma(\tilde{w}_i)} \]  

Albrecht & Bernstein 2006

\[ \text{FoM} = \left[ \sum_{i=1}^{N_{\text{bins}}} \frac{1}{\sigma^2(\tilde{w}_i)} \right]^{1/2} \]  

Sullivan, Sarkar, Joudaki, Amblard, Holz & Cooray 2007
How to parametrize modified gravity
How to parametrize modified gravity

How to parametrize modified gravity


2. Parametrize the expansion and growth history separately; check consistency
Beyond measuring $w(z)$, we can ask...

**Dark Energy or Modified Gravity?**

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi \rho_M \delta = 0$$

*Assuming smooth DE

- A given DE and modified gravity models may both fit the expansion history data very well
- But they will predict different structure formation history
- **Linear growth** is hard to compute even in fully well defined models for modified gravity (e.g. DGP)
- **Nonlinear growth** is much harder still to compute (c.f. this is a challenge even in GR!)
Strategy I: distance (z), growth(z) separately

Measure $r(z)$, $g(z)$, see if they agree

Knox, Song & Tyson 2005
Strategy II: \((\Omega_m, w_0, w_a)\) separately

Measure \(w_0\) and \(w_1\) for growth and distance, see if they agree

Ishak, Upadhye & Spergel 2005, others...
Strategy II.5: w separately, real data

Nice work, but current constraints are weak

Wang, Hui, May & Haiman, 2007
Strategy III: “Minimalist Modified Gravity”

\[ g(a) \equiv \frac{\delta}{a} = \exp \left[ \int_0^a d \ln a [\Omega_M(a)\gamma - 1] \right] \]

Excellent fit to standard DE cosmology with

\[ \gamma = 0.55 + 0.05[1 + w(z = 1)] \]

Linder 2005

• Gamma is a new parameter - the growth index - and we should measure it!

• E.g. fits DGP with value different from GR by \( \Delta \gamma = 0.13 \)

• For a moment, let us assume that the usual prescription for the nonlinear power spectrum is unchanged

• Apply to weak lensing and number counts; SNe and CMB remain unaffected

Huterer & Linder, astro-ph/0608681
Constraints on the growth index

<table>
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<tr>
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<th>sig($w_a$)</th>
<th>sig($\gamma$)</th>
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<tr>
<td>WL</td>
<td>0.33</td>
<td>1.16</td>
<td>0.23</td>
</tr>
<tr>
<td>+SNE</td>
<td>0.06</td>
<td>0.28</td>
<td>0.10</td>
</tr>
<tr>
<td>+Planck</td>
<td>0.06</td>
<td>0.21</td>
<td>0.044</td>
</tr>
<tr>
<td>+Clusters</td>
<td>0.05</td>
<td>0.16</td>
<td>0.037</td>
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WL: 1000 sqdeg (SNAP)
SNe: 2800 SNe (SNAP)
Clusters: 4000 sqdeg (SPT), dN/dz only, but mass-obs relation exact
parameters: $\Omega_d$, $A$, $w_0$, $w_a$, $\Omega_{mh}$, $\Omega_{bh}$, $m_{nu}$, $\gamma$

Huterer & Linder, astro-ph/0608681
Effects of discarding the small-scale info in weak lensing

Using the Nulling Tomography of weak lensing (Huterer & White 2005)
Effects of discarding the small-scale info in weak lensing

Using the Nulling Tomography of weak lensing (Huterer & White 2005)

- Clearly, errors increase dramatically as you keep only linear scales
- For MG, it’s hard to trust NL scales
- But for testing w(z) GR models, using NL scales may be possible
The data are now consistent with LCDM, but that may change.

If so, what observational strategies do we use to determine which violation of Occam’s Razor has the nature served us?

Possible alternatives:

- $w(z)$
- early DE
- curvature $\neq 0$
- clustered DE
- modified gravity
- more than one of the above
- .....
flat LCDM

$z < z_{\text{max}}$  $z > z_{\text{max}}$  $\Omega_K = 0$
$w = -1$  $w = -1$

add curvature

add $w(z)$

$z < z_{\text{max}}$  $z > z_{\text{max}}$  $\Omega_K = 0$
$w = -1$  $w = -1$

add curv and $w(z)$

$z < z_{\text{max}}$  $z > z_{\text{max}}$  $\Omega_K = 0$
$w = -1$

add w(z) and early DE

$z < z_{\text{max}}$  $z > z_{\text{max}}$  $\Omega_K = 0$
$w = w_\infty$

add curv, early DE and $w(z)$

$z < z_{\text{max}}$  $z > z_{\text{max}}$  $\Omega_K = 0$
$w = w_\infty$

add modified gravity

Early DE

Mortonson, Hu & Huterer, in preparation