

# NICMOS Distortion Correction

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## ABSTRACT

*The NICMOS distortion was measured by a series of images of the star cluster NGC1850. In each camera 5 images were taken, displaced from each other without rotation. The displacements known from the telescope movements were paired with corresponding movements of star images on the detector to calculate the solution. The degree of distortion was very small, amounting to about a one pixel shift at the corners. The pixel axes depart measurably from orthogonality being at most 0.2 degrees acute on camera 1.*

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## 1. Introduction

The distortion correction for the NICMOS camera is expected to be quite small because the area covered by each detector is small, less than 1 minute of arc square even for camera 3. However this needs to be confirmed and the actual degree of distortion calibrated for the purpose of accurately locating stars on the detector, and for the effect it may have on photometry due to the varying effective pixel area.

## 2. Measurements

Proposal 7040 took observations of the astrometric field NGC1850 in all three NICMOS cameras between May 19th and 25th, 1997 using filter F160W. Fifteen exposures, five positions per camera, were taken so as to have the same group of stars in each quadrant and in the center of the chips. The twelve offset pointings were offset from the central one by a quarter the FOV in x and y - 2.5 arcsec for NIC1, 0.5 arcsec for NIC2, and 12.5 arcsec for NIC3. For each camera, the five pointings were combined into a single mosaiced image. Stars were numbered and centroid positions found on this master image and then the STSDAS tasks *xy2rd* and *rd2xy* were used to locate the stars on each individual image. The *apphot center* task was used to calculate the final star centroid positions.

### 3. Analysis Methods

The (x,y) position of a star image on a detector may be related to the (V2,V3) position of the same star in the telescope field of view in two steps. First the (x,y) position is corrected for distortion to produce positions (xc, yc). These are then related to the (V2,V3) by a simple translation, scaling and rotation. The formulae used are

$$x_c = \sum_{i,j} a_{ij} x^{i-j} \cdot y^j \quad \text{and} \quad y_c = \sum_{i,j} b_{ij} \cdot x^{i-j} y^j$$

These are simple polynomials in which the terms where the sum of x and y powers is constant and equal to the order i have the same value of i.

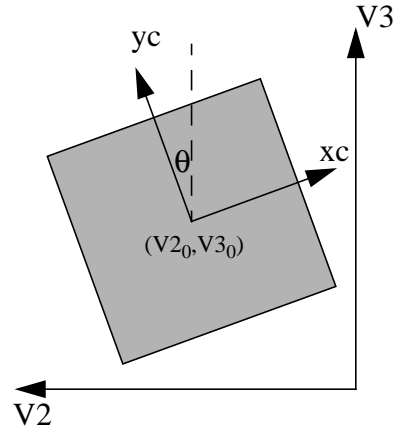
Explicitly

$$\begin{aligned} x_c &= a_{00} \\ &+ a_{10}x + a_{11}y \\ &+ a_{20}x^2 + a_{21}xy + a_{22}y^2 \\ &+ a_{30}x^3 + a_{31}x^2y + a_{32}xy^2 + a_{33}y^3 + \dots \end{aligned}$$

The transformation to the V2V3 frame is

$$V_2 = V_{2_0} - S_x \cdot x_c \cdot \cos \theta + S_y \cdot y_c \cdot \sin \theta$$

$$V_3 = V_{3_0} + S_x \cdot x_c \cdot \sin \theta + S_y \cdot y_c \cdot \cos \theta$$



It is convenient to choose the x,y and xc, yc origins to coincide which defines  $a_{00}$  and  $b_{00}$  to be zero. This coincident origin is taken to be at the center of the detector so that the fundamental scales and orientations refer to this point. By absorbing any scale changes into  $S_x$  and  $S_y$  we also fix  $a_{10}$  and  $b_{11}$  at 1.  $b_{10}$ , which is the coefficient of the y-dependence on x, is found by the fit but  $a_{11}$  is held at zero. This fixes the y-axis at angle  $\theta$  with respect to  $V_3$  but allows the direction of the distorted x-axis to depart from orthogonality with the y-axis.  $V_{2_0}$  and  $V_{3_0}$  represent the position of the chip center in  $V_2, V_3$ . Since we only deal with relative displacements, their actual values are not involved in the calculation. For the same reason, the actual positions of the stars do not enter into the calculation.

The relative values of  $V_2$  and  $V_3$  were derived from the displacements of the guide stars during the POS TARG displacements of each observation. There were 5 measurement positions for each camera. With respect to the initial position the other positions were displaced about half way along the four diagonal directions. The eight nearest-neighbor differences were used in the calculation. The displacement involving opposite

diagonal positions were omitted because they had few stars in common and would, in any case, be redundant.

For each pair of displacements, p and q, we calculate the difference

$$\Delta V2 = (V2_p - V2_q) - (-Sx(xc_p - xc_q) \cos \theta + Sy(yc_p - yc_q) \sin \theta)$$

$$\Delta V3 = (V3_p - V3_q) - (Sx(xc_p - xc_q) \sin \theta + Sy(yc_p - yc_q) \cos \theta)$$

$xc$  and  $yc$  incorporate the sought after coefficients. We minimize the statistic  $\Delta V2^2 + \Delta V3^2$ , summed over all measured combinations of star and pairs of positions. This was performed by a non-linear least squares search using the conjugate gradient method embodied in the Numerical Recipes routine FRPRMN. (Press et al. 1986). This method relies only on the measurements of the centroids of the stars and the motion of the telescope as determined by the Fine Guidance System.

A second analysis method was also employed, for reasons that are discussed in the results section. The non-linear minimization was necessary only to incorporate solving for the detector orientation. Once that is known, the x and y displacements can be handled separately and a linear fit performed on each. In terms of corrected pixels the displacement in the x and y directions are

$$\Delta x = \frac{(-\Delta V2 \cos \theta + \Delta V3 \sin \theta)}{Sx} \quad \text{and} \quad \Delta y = \frac{(\Delta V2 \sin \theta + \Delta V3 \cos \theta)}{Sy}$$

In terms of the polynomial coefficients

$\Delta x_{p,q} = \sum a_{i,j} \cdot (x_p^{i-j} \cdot y_p^j - x_q^{i-j} \cdot y_q^j)$  with a similar equation for y The problem reduces to a  $\chi^2$  minimization leading to the solution of a set of linear equations. The Numerical Recipes programs LUDCMP and LUBKSB were used for the matrix inversion.

### ***Differential Velocity aberration.***

Differential Velocity aberration can cause an apparent distortion effect and it might have been necessary to correct the x,y values. However the magnitude of this effect is only about  $10^{-4}$  times the pixel difference and therefore could only amount to about 0.02 pixels between the center and a corner pixel. This is considerably smaller than the measurement scatter and was not included.

## 4. Results

Using the general non-linear minimization, each chip was fitted with linear, quadratic and cubic models, the linear result providing the starting position for the quadratic fit etc. In each case the quadratic fit provided a clear improvement over the linear fit but the cubic fit did no better. The x-axis was not quite orthogonal to the y-axis, the difference being about 0.2 degrees for camera 1 but considerably smaller for the other cameras. The quadratic terms were of magnitude  $10^{-5}$  implying distortions at the corners of about 1 pixel

The search for the cubic fit was rather inconclusive, the results sometimes being inferior to the quadratic solution. It would seem reasonable that the cubic fit should be at least as good as the quadratic fit even if the cubic terms turned out to be zero. The difficulty is probably that the  $\chi^2$  surface is a function of 18 parameters, and the contribution of the sought after terms are of a size comparable with the scatter in the measurements. The minimization search can easily get caught in a local minimum which does not differ significantly from the absolute minimum. Searches for solutions with higher order terms did not find any improvement.

To investigate this further, the linear minimization was performed. The results from the first phase gave values for the overall scales and orientation which did not change with fitting order so it seemed valid to accept these as the best values as input to the second fit. Fits up to order 5 were calculated using the linear method and for cameras 1 and 2 the results were essentially identical up to third order. Although the coefficients are slightly different, the calculated distortions differ by less than 1/20 pixel throughout the CCD. A further fit was performed in which only the orientation was input, the scales being found afresh. Again no significant differences were found.

For camera 3, a fifth order fit was necessary to incorporate the extreme distortion caused by vignetting. There is probably no value in using this solution as camera 3 was not well focussed and distortion correcting the vignetted area is not really useful. We have developed a solution which excludes the data from the vignetted edge. By discarding points in the region  $y < 50$  the fit behaves as for the other cameras, significant improvement up to order 2 but none beyond that. For the NICMOS campaign, camera 3 will be brought into focus and the vignetting eliminated. A new set of distortion measurements are planned and at that time the fit will be redone and published as an addendum to this ISR.

Estimates of the errors on the fitted parameters were obtained by scanning the  $\chi^2$  surface in the region of the minimum. The change in each parameter which increased the  $\chi^2$  per degree of freedom by 1 is given as the error.

For each camera, the Jacobian was also calculated as  $\frac{\partial x_c}{\partial x} \cdot \frac{\partial y_c}{\partial y} - \frac{\partial x_c}{\partial y} \cdot \frac{\partial y_c}{\partial x}$ . This gives the apparent area of a pixel normalized to the area of a pixel at the (128,128) point.

The variation did not exceed 0.5%. Contour plots of x distortion, y distortion, total distortion and Jacobian values are given for each camera.

### ***Plate Scale Changes***

The dewar anomaly has pushed the cold-well and the cameras closer to the field-divider assembly and cold-mask, thus changing their effective focal-lengths. This change in focal length is reflected as a change in the plate scale at each camera. The amplitude of the change is inversely proportional to each camera's focal length. As the dewar has slowly relaxed, the cameras in the cold-well have been slowly (asymptotically) moving back towards their nominal positions. Figure 4 shows the X and Y plate scales in each of the 3 cameras from launch through day 323, which is November 19, 1997. The lines are the relative plate-scale measurements made from the focus monitor data, which is currently running every 2 weeks. The symbols show the absolute measures of the plate scale made from the 7039 NICMOS to FGS Alignment proposal which ran during SMOV. A report on plate scale changes will be issued separately.

The form of the solution is such that changing the plate scale does not require changes to other coefficients. The plate scales in the given solutions are as measured in May. The current plate scales and the history are published and will be maintained on the World Wide Web at

*[http://www.stsci.edu/ftp/instrument\\_news/NICMOS/nicmos\\_doc\\_platescale.html](http://www.stsci.edu/ftp/instrument_news/NICMOS/nicmos_doc_platescale.html)*

### ***NICMOS1***

x scale	0.043276 +/- 0.000006 arcsec/pixel
y scale	0.043110 +/- 0.000006 arcsec/pixel
y axis angle	-44.542 +/- 0.006 degrees
a <sub>20</sub>	(10.9 +/- 1.4) 10 <sup>-6</sup>
a <sub>21</sub>	(-10.6 +/- 5.9) 10 <sup>-6</sup>
a <sub>22</sub>	(7.4 +/- 2.6) 10 <sup>-6</sup>
b <sub>10</sub>	(-0.00485 +/- 0.00017)
	implying an x axis tilt of -0.277 +/- 0.009 degrees
b <sub>20</sub>	(5.0 +/- 1.4) 10 <sup>-6</sup>
b <sub>21</sub>	(10.7 +/- 4.5) 10 <sup>-6</sup>
b <sub>22</sub>	(14.4 +/- 2.1) 10 <sup>-6</sup>

The RMS deviation is 0.27 pixels or 11.6 milliarcsec

### ***NICMOS2***

x scale	0.076150 +/- 0.000006 arcsec/pixel
y scale	0.075498 +/- 0.000004 arcsec/pixel
y axis angle	-45.459 +/- 0.002 degrees
a <sub>20</sub>	(-9.98 +/- 0.85) 10 <sup>-6</sup>
a <sub>21</sub>	(-1.17 +/- 0.26) 10 <sup>-6</sup>
a <sub>22</sub>	(-4.45 +/- 0.71) 10 <sup>-6</sup>
b <sub>10</sub>	(-2.97 +/- 0.53)10 <sup>-4</sup> implying an x axis tilt of -0.017 +/- 0.003 degrees
b <sub>20</sub>	(1.02 +/- 0.82) 10 <sup>-6</sup>
b <sub>21</sub>	(-2.57 +/- 0.73) 10 <sup>-6</sup>
b <sub>22</sub>	(-0.15 +/- 1.27) 10 <sup>-6</sup>

RMS deviation is 0.18 pixels, 13.6 milliarcsec.

### ***NICMOS3***

x scale	0.204351 +/- 0.000015 arcsec/pixel
y scale	0.203584 +/- 0.000010 arcsec/pixel
y axis angle	-45.086 +/- 0.012 degrees
a <sub>20</sub>	(8.02 +/- 1.07) 10 <sup>-6</sup>
a <sub>21</sub>	(13.2 +/- 1.3) 10 <sup>-6</sup>
a <sub>22</sub>	(5.82 +/- 1.11) 10 <sup>-6</sup>
b <sub>10</sub>	(-8.95 +/- 1.26) 10 <sup>-4</sup> implying an x axis tilt of -0.051 +/- 0.007 degrees
b <sub>20</sub>	(-18.1 +/- 0.9) 10 <sup>-6</sup>

$$b_{21} \quad (0.6 \pm 1.0) 10^{-6}$$

$$b_{22} \quad (-11.6 \pm 1.2) 10^{-6}$$

RMS deviation is 0.30 pixels, 61.5 milliarcsec. Only the area for which  $y > 50$  is fitted.

A reminder about all of these solutions, they are based on an origin at pixel (128,128) To use the coefficients as given, first subtract 128 from the raw  $x$  and  $y$ , apply the distortion solution, and then add the 128 back to get to undistorted pixels. In terms of original pixels, the corrected pixels are  $xc = x + \sum_{i,j} a_{i,j} \cdot (x - 128)^{i-j} \cdot (y - 128)^j$  with  $a_{ij}$  replaced by  $b_{ij}$  for  $yc$ .

## 5. References

William H. Press, Brian P. Flannery, Saul A. Teukolsky, William T. Vetterling, *Numerical Recipes: The Art of Scientific Computing*, Cambridge University Press, 1986

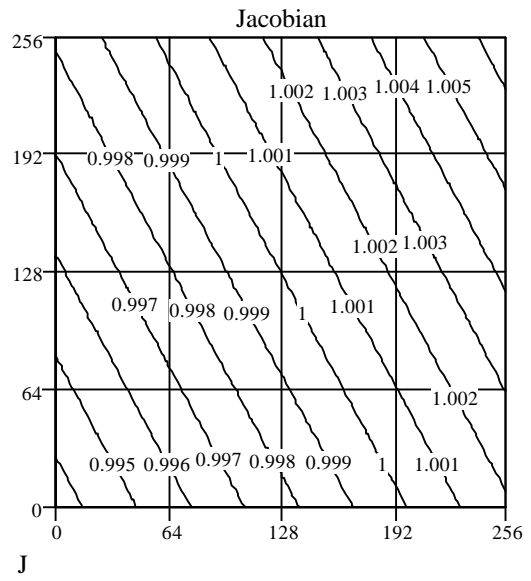
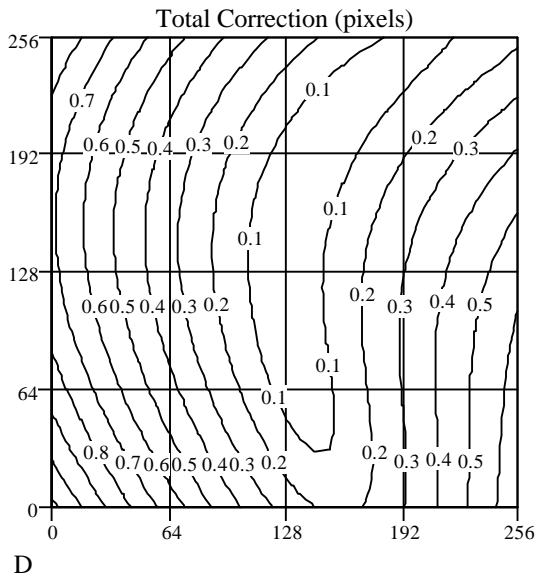
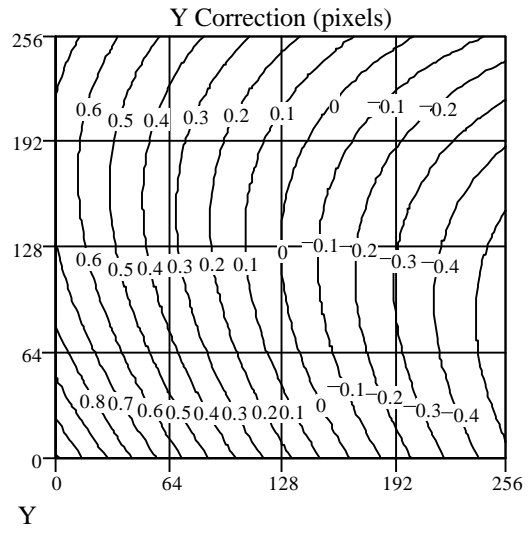
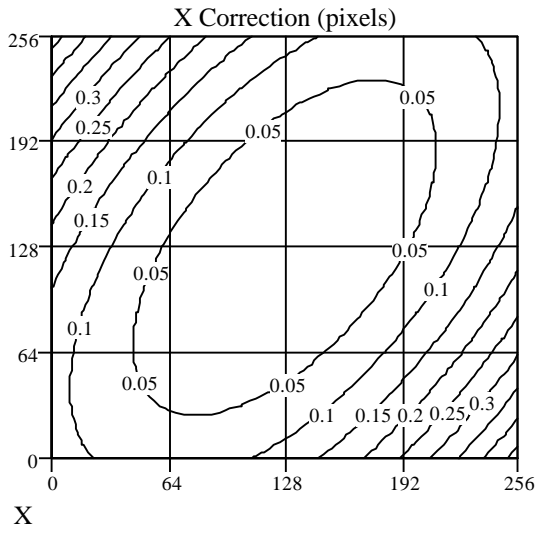


Figure 1: NICMOS Camera 1



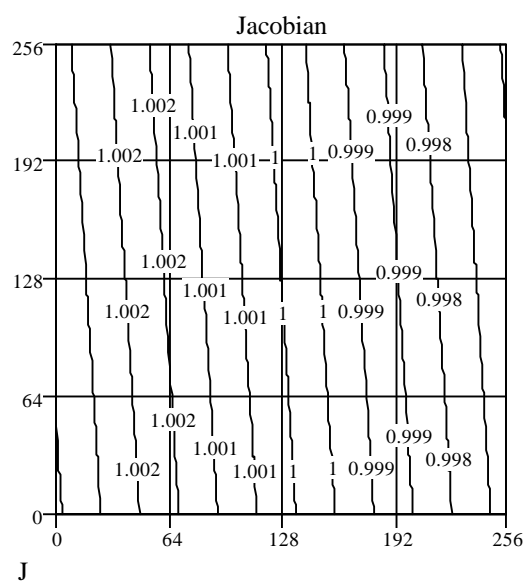
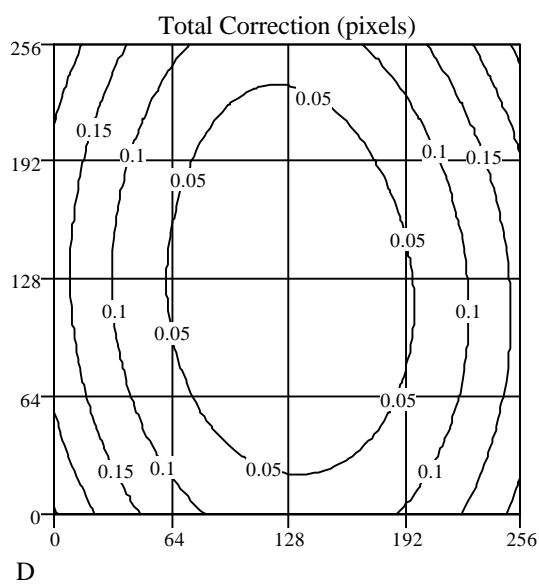
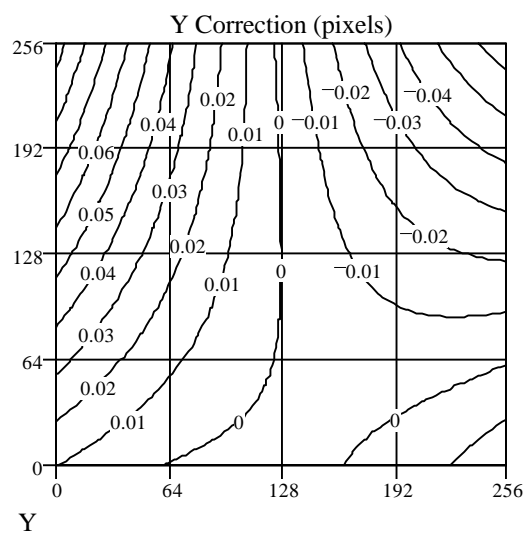
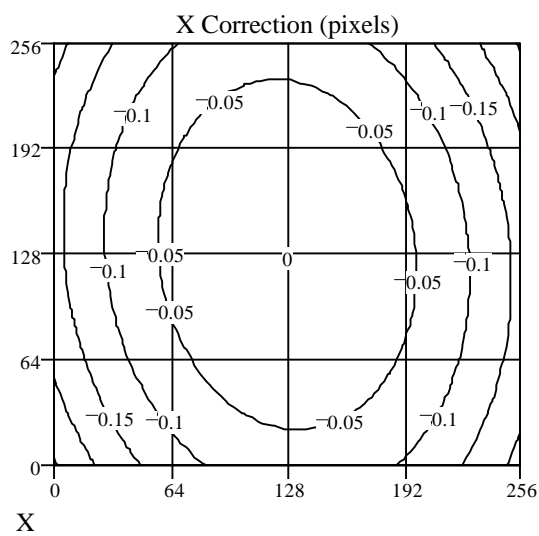


Figure 2: NICMOS Camera 2

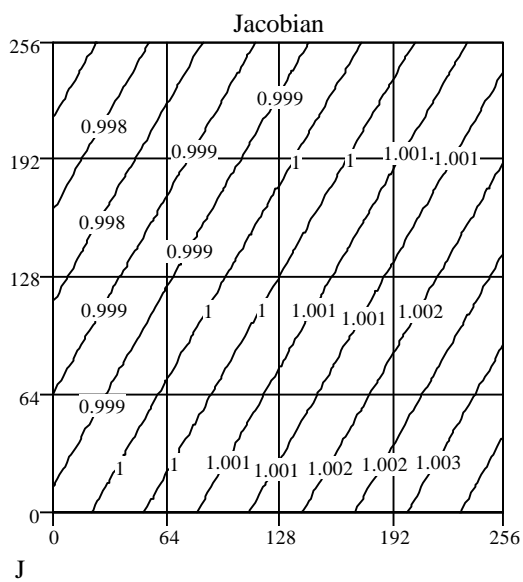
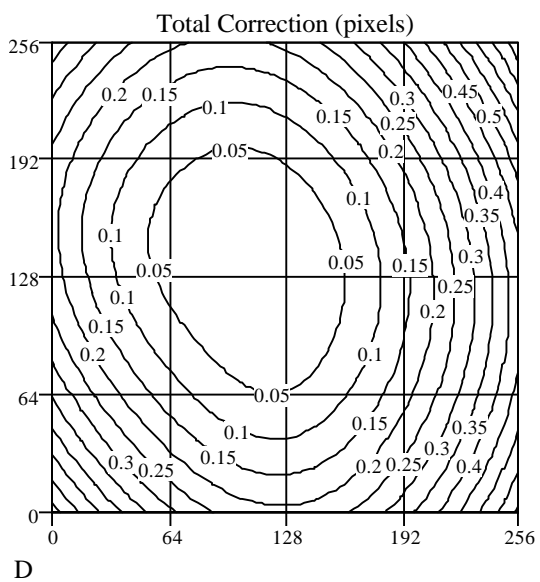
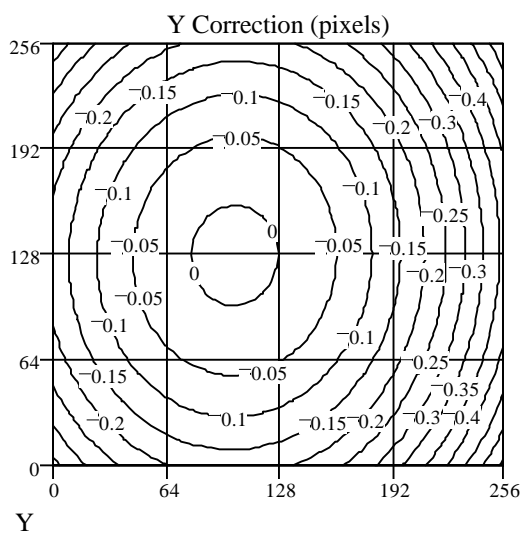
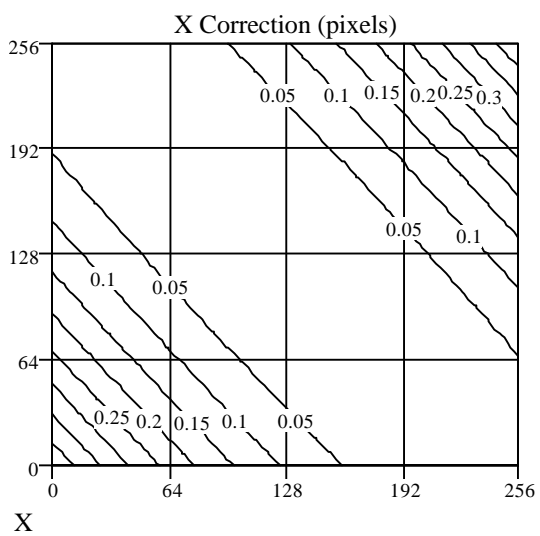


Figure 3: NICMOS Camera 3

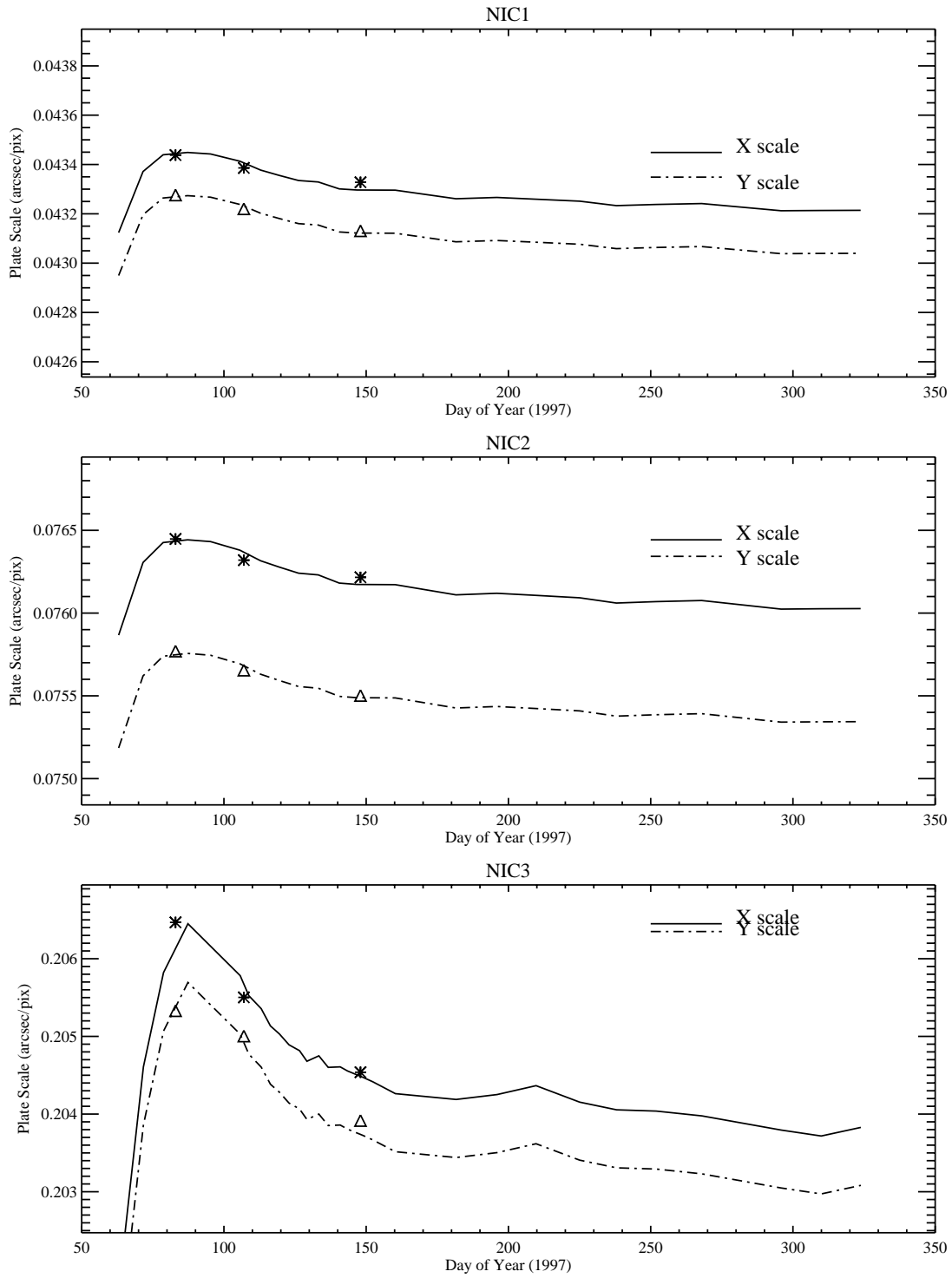


Figure 4: Temporal variation of plate scales