

The WFPC2 Scales and Alignments

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The WFPC2 distortion solution has been presented by Holtzman et al. in the WFPC2 SMOV report and submitted for publication (Holtzman et al.1994). This solution together with a WFPC2 observation of the open cluster M67 has been used to establish the scales and rotations with respect to the HST V2 V3 coordinate system.

The Holtzman solution relates pixel positions on each chip to a single "meta-frame" coordinate system (X, Y) which is nominally centered at the pyramid vertex and has pixels of the same size as those on PC1. The solution is expressed as a set of cubic polynomials in x_i and y_i , in which the origins of x_i and y_i are at (400,400) on chip number i .

$$X = C_{1i} + C_{2i}x_i + C_{3i}y_i + C_{4i}x_i^2 + C_{5i}x_iy_i + C_{6i}y_i^2 + C_{7i}x_i^3 + C_{8i}x_i^2y_i + C_{9i}x_iy_i^2 + C_{10i}y_i^3$$

$$Y = D_{1i} + D_{2i}x_i + D_{3i}y_i + D_{4i}x_i^2 + D_{5i}x_iy_i + D_{6i}y_i^2 + D_{7i}x_i^3 + D_{8i}x_i^2y_i + D_{9i}x_iy_i^2 + D_{10i}y_i^3$$

The connection to V2 V3 is a linear transformation from the meta-frame involving a single overall scale, a translation and a rotation.

This is expressed as:

$$V2 = V2_0 + s[-X \cos(\theta) + Y \sin(\theta)]$$

$$V3 = V3_0 + s[X \sin(\theta) + Y \cos(\theta)]$$

so that $(V2_0, V3_0)$ corresponds to the origin of the X, Y system, s is the scale, θ represents the rotation from $V3$ to the Y axis, and a parity change between (X, Y) and $(V2, V3)$ has been incorporated.

The values of (x, y) are the centroids of M67 stars in the observation U22U0501 made on January 21st 1994. The centroids were measured by Ellyne Kinney. The stars were matched by Ron Gilliland to a set of astrometric measurements described in Girard et al.(1989). The star information is given in Table 1. The star numbering matches a more recent survey (Montgomery, Marschall and Janes, 1993) which includes those from the earlier survey plus many fainter stars.

Using the tabulated Right Ascension and Declinations of the stars and Fine Guidance System information, accurate $V2$ $V3$ positions were calculated for each centroided star. Each (x, y) was transformed to an (X, Y) value and a linear least-squares fit performed to extract the values of $V2_0, V3_0, s$ and θ . The origin $(V2_0, V3_0)$ evaluates to (1.80, -7.03) arcseconds. The value of s was found to be 0.04556 +/- 0.00002 arcsec/pixel and $\theta = 224.74$ +/- 0.03 degrees. To obtain the scales and rotations in each detector we must combine this result with details of the distortion solution. The actual values vary slowly across the detectors and, therefore, we must choose the positions at which to define scales and rotations. The scale along a detector x axis is $\sqrt{(\partial X/\partial x)^2 + (\partial Y/\partial x)^2}$ with a similar result for the y axis. At the (400,400) point on each detector these are $\sqrt{C_{2i}^2 + D_{2i}^2}$ and $\sqrt{C_{3i}^2 + D_{3i}^2}$. The angles of the axes are also defined in terms of the differentials. All angles

are measured from the Y axis in an anti-clockwise direction and therefore may be added to the V3 angle defined above to give the final directions with respect to V3. For the x axes the tangent of the slope is $-(\partial X/\partial x)/(\partial Y/\partial x)$ and the yaxis tangent is $-(\partial X/\partial y)/(\partial Y/\partial y)$. Again at the (400, 400) positions these are simply $-C_{2i}/D_{2i}$ and $-C_{3i}/D_{3i}$.

Table 1: Star Information

Detector	Star Number	x (pixels)	y (pixels)	V2 (arcsec)	V3 (arcsec)
PC1	5764	101.10	109.18	1.910	-10.636
WF2	5741	607.72	110.15	-42.834	-40.565
WF2	5781	352.86	304.10	-38.939	-9.077
WF2	5803	350.09	473.12	-50.816	2.859
WF2	5807	751.69	720.65	-95.983	-8.678
WF2	5813	407.15	603.08	-64.045	7.761
WF3	5788	115.31	178.84	5.169	8.358
WF3	5820	173.01	682.42	36.138	47.734
WF3	5844	499.41	528.49	2.339	59.852
WF3	5847	439.22	717.68	19.775	69.073
WF4	5760	632.12	423.03	69.617	7.710
WF4	5763	657.88	425.87	71.502	9.558
WF4	5768	503.45	309.41	52.507	7.032
WF4	5777	228.99	64.18	16.047	4.935
WF4	5790	529.01	141.93	42.556	20.553

The raw results showing the relationship between the chips and the meta-field are shown in Table 1. These have been evaluated, not at the (400,400) position, but, at the detector reference points which are (420, 424.5), (423.5, 414), (436.5, 424.5) and (423, 421) for PC1, WF2, WF3 and WF4 respectively.

Table 2: WFPC2 Detector related to meta-frame

Detector	x scale (relative)	y scale (relative)	x angle (degrees)	y angle (degrees)
PC1	1.000690	0.999618	270.05	-0.05
WF2	2.185618	2.187730	-0.51	89.47
WF3	2.187240	2.185178	89.80	179.78
WF4	2.186485	2.188585	180.36	270.34

Combining the above results with the transformation between the meta-field and V2 V3 gives the final scales and rotations.

Table 3: Scales and Rotations related to V2 V3

Detector	x scale (arcsec/pixel)	y scale (arcsec/pixel)	x angle (degrees)	y angle (degrees)
PC1	0.045561	0.045541	134.80	224.70
WF2	0.099573	0.099669	224.24	314.22
WF3	0.099647	0.099553	314.55	44.52
WF4	0.099612	0.099708	45.11	135.09

The x and y angles have been calculated independently, but the x angle is expected to be equal to the y angle plus 90 degrees, (modulo 360). This is true within the estimated errors. For accurate positional calculations the Holtzman solution should be used and any slight shear distortion is automatically included. For the numbers to be entered in the Project Data Base, which are principally used for calculating telescope movements offset from the reference points, it is probably preferable to avoid this complication, keep the axes strictly orthogonal and use mean values based on the pair of numbers. The scale differences along x and y axes appear to be statistically significant and should be retained. We have accepted this asymmetry in the PDB before and no special problems have arisen. So the final set of values to be placed in the Science Instrument Aperture File should be as given in Table 4.

Table 4: Aperture Parameters

Detector	Reference Pixel		Reference Point (arc seconds)		Scales (arc seconds/pixel)		Axis angle (degrees)	
	x	y	V2	V3	x	y	x	y
PC1	420.0	424.5	-0.42 *	48.01 *	0.045561	0.045541	134.75	224.75
WF2	423.5	414.0	51.69	-6.49	0.099573	0.099669	224.23	314.23
WF3	436.5	424.5	-0.42	48.01	0.099647	0.099553	314.54	44.54
WF4	423.0	421.0	54.70	-6.60	0.099612	0.099708	45.10	135.10

A concern has been expressed that changing V2 V3 positions now will complicate relating earlier observations to forthcoming ones. It is true that a target will be placed at a slightly different position, the magnitude of the shift being about 0.3 arcsec. I consider it more important to provide the best current knowledge in the PDB but also retain the history so that observations taken many months apart may be related. Note that the scales have increased by about 1% and it may be important to reflect this in the PDB so that positions of objects well away from the center are correctly calculated.

* Note: The document, as originally published, lists the values -0.42 and 48.01 as PC reference points in V2 and V3 respectively. The correct values for this epoch should be 2.09 in V2 and -30.95 in V3.

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