TECHNICAL REPORT

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Title: Maximum Allowable Solar Torque Architecture for JWST		TM: JWST-STScI-000583 SM-12 Date: May 10, 2005 Rev.: -
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1.0 Abstract

In FY04-05, the STScI has completed five studies on angular momentum management. This report compares the results of these studies and identifies the maximum allowable solar torque architecture for JWST. This dimensionless parameter, *P*, is given by a characteristic time (~ sqrt(minimum days between dump*effective visit time in days)) multiplied by the appropriate RMS sunshield torque and divided by the effective wheel capacity. For parameter values $P > 0.5$, the STScI cannot reliably execute the nominal JWST mission at 95% scheduling efficiency while limiting the angular momentum buildup. Even at that level, the STScI must develop procedures and software tools achieve high scheduling efficiency while managing JWST angular momentum and station-keeping fuel. The most recent torque tables, which have higher torques than reported in 2004, coupled with the potential ability to dump every 11 days and run the reaction wheels through zero, result in an architecture parameter, $P \sim 0.43$, that lies just below the recommended upper limit.

2.0 Introduction

In the last year, the STScI has studied the impact and requirements of angular momentum management in the planning system. Initially, these studies showed that the ASWG DRM frequently exceeded the 24 N-m-s limits in less than the 22-day station-keeping period (Kinzel 2004). That study has been repeated periodically as the STScI receives new torque tables made available by NGST. Petro et al. (2005) have done a similar analysis of the high fidelity SO-DRM. Another study considered the angular momentum build-up as a random walk (Stockman & Long, 2004). Two studies have considered the angular momentum buildup using Monte Carlo methods (Kinzel 2005, Long 2005). Since these studies have used different assumptions concerning the average visit time, the range of roll angles permitted, and JWST torque tables, it is not easy to compare their results. This report compares the five studies mentioned above and places them on a common footing. With this information we recommend a maximum allowable solar torque architecture for JWST.

3.0 Model Comparisons

3.1 The Random Walk Model

Using editorial privilege and to create common definitions for the important parameters related with angular momentum management, we start with a review of the random walk model. This model and the three that follow begin with the assumption that the JWST mission is comprised of uncorrelated visits, approximately uniformly distributed on the celestial sphere. In this case, we can model the build-up of angular momentum as a random walk in the 2-D space of pitch and roll angular momentum in a fixed inertial frame. The 2004 random walk study ignored the slow mixing of momentum in roll and pitch due the JWST L2 orbit and the build-up "propeller" momentum[1]: the buildup of an angular momentum vector toward the Sun due to sunshield asymmetries or the projection of body pitch or roll into this axis for off-nominal pitches (± sin 22.5°). Analysis of simulated mission schedules generally treat angular momentum in a true 3-D inertial frame.

In the random walk model, the average angular momentum buildup, $<L>$, for n visits is given by

$$<L> \sim \mathrm{sqrt}\,(n)\ <t_{visit}>\ <l>\ , \qquad (1)$$

where $<t_{visit}>$ is the RMS average duration of a visit and $<l>$ is the RMS torque during a visit. When L is comparable to the JWST reaction wheel capacity, the S&OC must plan to dump momentum at some time during the n visits. The study showed that the average maximum buildup during the n steps was 1.21 times the average value at the end of all the steps (e.g. the end point is usually a little closer to the origin than the maximum value reached during the sequence. The study also showed that the maximum radius reached during the walk is less than 2 x the RMS average in 97% of the trials.

To further refine the architecture parameter we need to consider the effects of different assumptions on the distribution of visit lengths. The average quantities <t> and <l> are defined in the context of a random walk and are, in general, RMS averages of distributions in time and momentum input. In terms of the JWST mission, the random walk study assumptions were:

Number of visits per year: 2000

Distribution of visits: Log normal with a cutoff at 1000 sec. (Petro, 2004)

Effective random walk time per visit: $t_{rms} \sim 0.38$ d (note that the log-normal distribution results in an RMS visit length considerably longer than 365 d/ 2000 ~ 0.18 d)

Station keeping interval: 22 days

Angular momentum dumping interval: 22 days

Wheel capacity: 40 N-m-s (6-wheels), 24 N-m-s (4-wheels)

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[1] "Yaw" or "propeller" angular momentum accumulation in the JWST body frame is ~ 4 times less than that in pitch and roll. But it can mix into the other two frames with off-nominal pitch angles. As a source of noise, it should aggravate the angular momentum accumulation. However, this mixing can also serve to reduce the amount of pitch body frame angular momentum that builds up the in the inertial pitch and roll axis (momentum vectors perpendicular to the sun line.

Restricted room-mean square pitch and roll build-up (0° – 45° in pitch, ± 1° in roll):

$L_{rms\_restricted}$ ~ 2.8 N-m-s/day Target locations: isotropic

We can recast equation 1 as

$$<L> \sim sqrt\, (t_{\mathrm{dump}}\, t_{\mathrm{visit}}) <\mathrm{l}> , \qquad (2)$$

and use an equivalent constant visit time, $t_{visit} = < t^{2}>/ < t > \sim 0.8$ days, to account for the statistics of the log-normal distribution in Petro (2004). Using equation 2 we can derive the angular momentum architecture parameter. P, given by equation 3.

$$P = sqrt\, (t_{\mathrm{dump}}\, t_{\mathrm{visit}}) <l> / L_{\mathrm{wheel}} \qquad (3)$$

Given the results of the random walk study (97% of trials have maximum angular momentum accumulations within 2 x the average), we can see that this parameter should be < 0.5 to manage angular momentum in a straightforward manner. When $P > 0.5$, the architecture cannot support the mode of scheduling being studied (too much torque, visits too long, wheel capacity too small, etc.).

The Stockman& Long (2004) angular momentum study argued that that RMS average of the rate of angular momentum build-up in pitch and roll in the 2004 NGST torque tables is the appropriate weighting for $\mathrm{l}_{\mathrm{eff}}$ in equations 1-3. If the full range of the 2004 sunshield were utilized, then the RMS pitch and roll torque, <l>, would be 5.6 N-m-s/day; if it could be limited to (0-45°) in pitch and ±4° in roll, the RMS torque would be 4.7 N-m-s/day, if 0-45° and ±1° it would be 2.8 N-m-s/day using the sunshield design (whichever one). The JWST operations concept currently requires the full field of regard of JWST, including the CVZ both for science observations and for calibration. However, computing the RMS over a pitch angle of 0-45° recognizes the fact that the solid angle occupied by the CVZ is relative small compared to lower pitch angles. Restricting the roll angle to ± 4° for the purposes of estimating the RMS is also reasonable since the scheduling algorithms will favor scheduling in this band to limit tight timing requirements and avoid straying outside the ±5° operational band. Restricting the roll angle to ±1° is an extreme situation, and would have significant implications for both the difficulty of scheduling and the time the Observatory could fly through “acquisition failures”, but we have considered it in the event there are no reasonable alternatives to the momentum problem.

In the list of assumptions, we provide an RMS torque averaged over ± 4° in roll (the 2004 study did not consider using the most extreme roll or pitch angles routinely) and an RMS torque averaged over ± 1°. The former average is appropriate for planning scenarios on which science constraints are important and override concerns about angular momentum buildup (except see scrambling study below). The narrower roll range is more appropriate for operations scenarios in which management of angular momentum is paramount and observations can be forced to execute near their nominal roll orientation (larger roll angles come with correspondingly high solar torques). The implications of such a small roll angle range have not yet been determined, but it would have effects on the amount of oversubscription required to effectively utilize the observatory and on the amount of time the Observatory can be allowed to "fly through" target acquisition failures.

The random walk study also considered the impact of constraining observations by science links, orient constraints, and reducing natural backgrounds. For a 50% constrained schedule, the average buildup of angular momentum in 22 days was ~ 15 N-m given the assumptions above and a modest momentum management scheme (scheduling approximately half of the visit pool as unconstrained observations between ±1° roll). In terms of the architecture parameter derived for this part of the study, it was equivalent to 3.8 N-m-s day * sqrt (22*0.8) / 30 N-m-s = 0.53, where 30 N-m-s is the required storage capacity to support reliable scheduling at this level. This result agrees with P ~ 0.5 criterion *by definition.* The remainder of this memo compares the results from the other studies for consistency given the assumed operations concepts. If they do, we will have confidence in deriving an architecture parameter for the JWST mission.

3.2 DRM Dump Frequencies

The STScI (Kinzel, 2004,2005) studied the ASWG Design Reference Mission (V3p6b), which was developed without consideration of angular momentum minimization. There are many long visits in this DRM, more than in the nominal JWST mission. For purposes of analyzing the angular momentum buildup, these long visits were divided into 1-day segments, with the roll re-zeroed to nominal at the mid-point for each segment. Table 1 gives the results for this study:

Angular Momentum Capacity	40 N-m-s			24 N-m-s		
Torque Table	Sep 03	Sep 04	Mar 05	Sep 03	Sep 04	Mar 05
RMS Torque (0°)	1.46	3.1	3.1	1.46	3.1	3.1
Dump Intervals (days)						
Mean	67.8	27.9	19.9	36.1	14.4	10.3
Minimum	19.5	5.7	9.5	11.0	2.6	5.4
Maximum	212.5	91.0	47.1	129.9	45.7	34.8
Number of Dumps	24.0	59.0	82.0	45.0	115.0	161.0
Predicted mean dump interval from random walk (days)	53	12	12	19	4.2	4.2

Table 1: The calculated dump frequencies and architecture parameters from Kinzel (2004,2005)

Here Kinzel assumed that the scheduling system is scheduling the observations at their nominal roll and at all valid pitch angles. We calculate the predicted mean dump interval based upon eq. 2 (times 1.21). We note that there are several entries where the minimum dump interval is less than half of the mean. We believe that this is due to the long visits at a given pitch (some exceed ten days) and that these long visits are not typical of the actual JWST mission. The average times per observation in the ASWG DRM (1.32 d mean, 3.62 d RMS average, 9.9 day effective time) are significantly longer than assumed in the random walk analysis assumptions (0.18 d mean, 0.38 d RMS, 0.8 d effective time). Given the size of these visits, the random walk model is probably not valid. Some dumps occur in the middle of long visits. Equation (2) is clearly wrong when the characteristic time is longer than the time between dumps. Some visits are so long that they average out variations in pitch. Nevertheless, there is qualitative agreement in predicted mean dump intervals and the mean dump interval in the analysis. Clearly the use of a specific DRM limits our ability to predict actual performance. Note that despite the same RMS torques for the 2004 and 2005 torque tables (for 0° roll), the mean dump intervals differ. This may be because of strong off-nominal roll torques at the beginnings or ends of visits in the 2005 analysis or because of the layout of the specific observations versus the torque tables.

3.3 SO-DRM Analysis

Petro et al (2005) have analyzed the angular momentum build-up in the high fidelity Science Operations Design Reference Mission (SO-DRM). "The SO–DRM is comprised of 4,066 visits, the duration of which range from 0.8 hr to 290 hr, with only approximately a half–dozen exceeding one day. In the present version of the SO–DRM visits longer than 1 day are used to simulate the effects of observations that must be conducted at a fixed orientation of the observatory. Scientifically required linkages between visits are implemented, as are time–constrained windows." (Petro et al 2005). The analysis covered the first 574 days of the schedule, during which gaps between visits were less than 1 day. Petro et al (2005) calculated the accumulated angular momentum during this period to be 954 N-m-s using the September 2004 torque tables. Using the random walk model, we would predict a build-up of approximately 86 N-m-s over 574 days, with an RMS torque of 4 N-m-s/day and a characteristic time of 0.8 days (the SO-DRM visit distribution is similar to the Petro (2004) distribution). Clearly the SO-DRM results cannot be explained by the random walk theory.

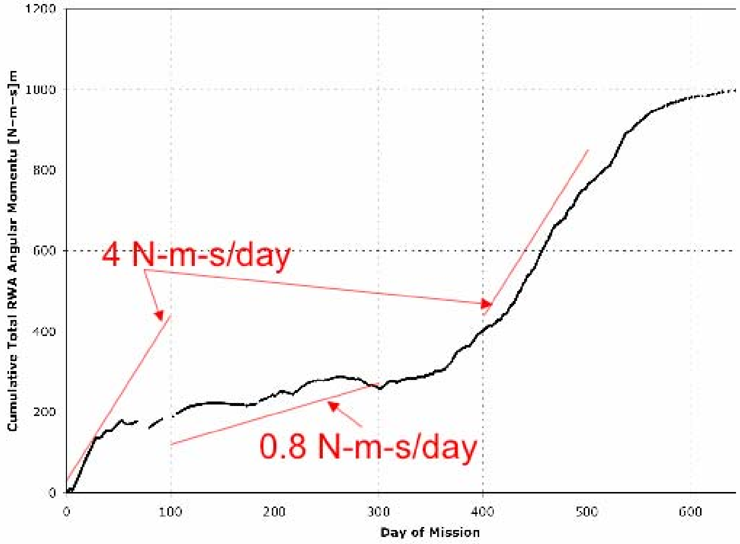

Figure 1: Cumulative total RWA angular momentum for the JMS mission schedule of the SO–DRM. Momentum unloading is not modeled. **(Petro et al 2005)**

In Figure 1, we reproduce figure 5 in Petro et al (2005) that shows the accumulation of angular momentum versus time. We note that the angular momentum does not increase as the square root of the time but has two sharp linear sections: one at the start of the interval and another beginning approximately one year after the start of the interval. We believe that each of these features is due to the "greedy" scheduling algorithm attempting to schedule its high difficulty (long and linked) observations as soon as possible. Such an algorithm will tend to create a strongly correlated schedule, favoring one side of the annular field of regard (FOR) over another. In that case, we would expect a strong linear build-up of angular momentum at approximately 4 N-m-s day or greater. We show the slope of such a build-up in Figure 1 compared to these two linear regions. We also note that there are long stretches of the calendar in which the slope equivalent to a 22-day build-up (0.8 N-m-s/day) is appropriate. Hence we believe that a scheduling algorithm that favored more distributed scheduling over the FOR would achieve the performance predicted by the random walk model.

3.4 Monte Carlo Simulations

Long (2005) examines the affects of oversubscription, orient constraints, and scheduling algorithms on the efficiency of scheduling and the build-up of angular momentum. The study assumes average visit lengths (of a log normal distribution) of 20,000, 40,000 and 80,000 s. These correspond to effective times of 0.50d, 0.99d, 1.98d in a random walk study. To understand the impact of restricting the scheduling of observations to minimize overall angular momentum, the study is restricted to 40,000 s observations (0.99 days effective time). Long finds that by restricting the angular momentum per visit to approximately 2.0 N-m-s, two very different scheduling techniques can reduce the number of dumps required by approximately a factor of two compared to unrestricted scheduling. In particular, Long finds that the number of dumps required per year assuming an angular momentum capacity of 22 N-m-s is approximately 14 dumps per year (we have corrected by the scheduling efficiency) if one assumes zero roll and a maximum torque in pitch of approximately 2 N-m-s/day. Like the SO-DRM scheduler, Long's scheduling technique tends to choose observations as soon as they are feasible (in the FOR) and builds up angular momentum linearly with time, rather than as the square root. In the extreme case, the schedule would build up angular momentum at ~2 N-m-s/day and require dumps every eleven days (or 33 per year). If approximately half of the schedule had such a trend, it would explain the higher dump rate.

We have also examined the biasing effects of scheduling near the extremes of the pitch ranges (near -5° and +45°). In such a case, the appropriate weighting of the torque tables is one that emphasizes the pitch extremes. To provide an estimate of the end-weighting effect, we have analyzed the September 2004 torque tables with a linear ramp weighting over 5 and 10 degrees at the pitch extremes. The results are shown in Table 2 for ±1° and ±4° roll ranges. The biggest effect, approximately a factor of two compared to uniform pitch weighting, is for the narrower roll range. This is because the RMS torque for the wider roll range is dominated by roll rather than pitch.

Roll Range	Uniform Weighting N-m-s/day	10° End Weight N-m-s/day	5° End Weight N-m-s/day
±1°	3.26	4.76	5.24
±5°	4.85	5.36	5.37

Table 2: RMS Torques from the September 2004 Torque Tables

3.5 Visit Sequencing

In Kinzel (2005), he studies the effects of visit sequencing and roll angle selection to minimize angular momentum in given 22 day Observation Plans. Visits are assumed to be 1 day long. The September 2004 torque tables are used. The targets are assumed to be roughly isotropic in the field of regard except for the constraint that the targets be visible throughout the 22-day interval. Targets near the ecliptic plane tend to be observed at moderate pitch angles – e.g. say 11 days from either pitch extreme. Ignoring this last caveat, we can compare the results of the initial model with the angular momentum parameter described above. Kinzel finds that the mean angular momentum after 22 days is approximately 15 N-m-s. Given that Kinzel only included targets that could be observed for the entire 22-day period, we estimate that the RMS torque at zero roll is approximately 2.5 N-m-s/day. Equation 2 yields 11.72 N-m-s for the average angular momentum accumulated after scheduling 22 one-day visits. The typical maximum during the sequence is about 1.21 times larger than the mean or 14 N-m-s, more or less in agreement with equation 2. This is independent evidence that the random walk theory provides an accurate estimate of the build-up of angular momentum, given a scheduling system that randomly samples a given ranges of pitch and roll.

4.0 Summary

We have compared the results from five studies. We have found that the agreement with the simple random walk model is highly dependent on the scheduling algorithms used in those studies. If the algorithm is unbiased in its choice of targets in the FOR and the population of the pitch and roll range, the agreement is excellent (Kinzel 2005). However scheduling biases that achieve other goals such as scheduling targets when they are first accessible, can build-up torques more rapidly than a random scheduler. Only the Kinzel ASWG DRM study found dump rates slower than that predicted by the random walk theory. The failure of the random walk study in this case may be explained by the extremely long observations in the ASWG DRM creating a characteristic visit time, ~ 10 days, that causes the observation to traverse a large range of pitch and is comparable to the time between dumps.

Given our improved understanding of the operational capabilities needed to manage angular momentum, we are in a position to recommend maximum angular momentum architecture parameters based upon specific operations concepts. One such concept assumes that 50% of visits are science constrained and may thus require large off-nominal rolls and 50% of the visits may have their rolls set during short-term scheduling. In such a case, we would recommend that the architecture parameter for JWST be bounded by:

$$P = Sqrt\,(0.8\, t_{\mathbf{dump}}\,)\; *\; sqrt(\,0.5\,(<l^2_{\mathbf{+-4}}> + <l^2_{\mathbf{+-1}}>\,)\,/\,L_{\mathbf{wheels}} < 0.5 \qquad (4)$$

Such a bound would have roughly a factor of two margin of safety because of proven ability to reduce angular momentum in a given sequence by re-ordering and selecting roll angles (Kinzel 2005). Such a margin will be required, however, because of likely correlation effects in the long-range plan and short range scheduling system that can make the angular momentum grow with a component that is linear in time.

In terms of the March 24 2005 torque tables, the potential ability to run the reaction wheels through zero, and the potential of having 11 day dump intervals, the current JWST architecture parameter for a scheduling system that evenly samples the entire pitch and roll range (-5° to +45° in pitch and ±5° in roll) is given by:

$$P = Sqrt\,(0.8\ day * 11\ day) * 7\ (N\text{-}m\text{-}s/day)\,/(2 \times 24\ N\text{-}m\text{-}s) \sim 0.43 \qquad (5)$$

This value would be close to our nominal value of $P = 0.5$ for straightforward angular momentum management with minimal scientific impact. It would provide the S&OC a factor of two margin if the capability of re-ordering observations during short term scheduling were developed. Such a margin is important for setting requirements on the long range planning and short range scheduling systems (their performance will not be perfect) and because of the uncertainty in science mission assumptions such as the actual distribution of JWST targets and visit durations.

Given a satisfactory spacecraft architecture, we can similarly derive requirements on the planning and scheduling system of the following form:

4.1.1 Long Range Plan: Assuming the canonical science program (TBD), the LRP shall be provide a list of targets that evenly samples the field of regard in any 22-day period. The performance measure shall be that any correlated sampling represents less than 2.5 days (TBR) of the 22 or the average angular momentum increase over that period is less than 150% that predicted for an uncorrelated sample.

4.1.2 Short Range Scheduling: The short range scheduler shall have the capability of rearranging the order and roll orientations of unconstrained visits during a specified scheduling period (e.g. the minimum time between dumps or station keeping to minimize the maximum use of spacecraft resources (e.g. angular momentum).

5.0 References

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