Multipole analysis of gravitational wave recoil

Jeremy Schnittman¹, Alessandra Buonanno¹, James van Meter², John Baker², William Boggs¹,
Joan Centrella¹, Bernard Kelly³, and Sean McWilliams¹

(¹) University of Maryland; (²) NASA Goddard Space Flight Center

Abstract

We present a multipole analysis of recent numerical simulations of the inspiral and merger of black hole (BH) binaries, for both unequal masses and non-zero, non-processing spins. We show that the linear momentum flux from an inspiral phase moment can be calculated from the complex scalar \( F_\mu \), measured at large distance from the source:

\[
F_\mu = \frac{dP_\mu}{dt} \left( \frac{R}{2} \right) \left( \frac{dI}{dt} \right) \int \frac{dV}{dI} \mu dV.
\]

(1)

This flux can also be written as an expansion in the radiation multipole moments \( F_{\mu\nu} \) and \( S_{\mu\nu} \) (Thorne 1980). In the special case of non-rotating, nearly circular orbits, the flux can be written

\[
F_\mu = \frac{2 \pi G M^2}{c^2} \left( \frac{\mu}{R} \right)^2 \left( \frac{d\mu}{dR} \right) \frac{dR}{dt} \left( \frac{dR}{dt} \right).
\]

(2)

where in the last line we keep only the dominant contributions, while all other terms are typically smaller by at least an order of magnitude. We find that including all terms with \( f < 4 \) gives agreement with Eq. (1) within 1%, and just the three leading terms provides agreement. For each mode, we determine a dominant frequency as in Buonanno, Cook, & Pretorius (2007).

Amplitudes (left) and frequencies (right) of the dominant multipole modes for a non-spinning binary with \( \mu = 1 \). The relatively small amplitudes of the \( f_{\mu\nu} \) modes make it difficult to extract a meaningful frequency. The amplitudes increase through the inspiral phase and then decay exponentially at constant frequency during the ringdown phase. The time \( t_{1/2} \) corresponds to the time of peak GW energy flux.

For non-rotating binaries, the three dominant flux terms in Eq. (2) can be written to leading order as

\[
F(S_{\mu\nu}) = \frac{G M^2}{c^2} \left( \frac{\mu}{R} \right)^2 \left( \frac{d\mu}{dR} \right) \frac{dR}{dt} \left( \frac{dR}{dt} \right),
\]

(3)

where in normalized units with \( C = M = c = 1 \), \( S_{\mu\nu} \) is the contribution from spins parallel to the angular momentum. Lastly, we define the angles between these fluxes as

\[
\cos \theta^3 = \frac{F(S_{\mu\nu})}{F(\mu_\nu)}, \quad \cos \theta^3 = \frac{F(S_{\mu\nu})}{F(\mu_\nu)}, \quad \cos \theta^3 = \frac{F(S_{\mu\nu})}{F(\mu_\nu)},
\]

(4)

where on the right hand side we reduce to the ringdown modes with frequency \( \omega_0 \) defined above.

Anatomy of the Kick

Here we compare the results from two numerical simulations, both with \( \mu_1/\mu_2 = 1/2 \) and \( \omega_1/\omega_2 = 0/0 \), but with the spins oriented in opposite directions along the axis perpendicular to the orbital angular momentum. In the left column, the spins are pointed directly away from the source, giving a perfect anti-kick in the left column, the spins are pointed directly away from the source, giving a perfect anti-kick in the right column. The right column shows the anti-kick produced by the \( F(\mu_\nu) \) flux, and thus the \( \mu_\nu \) flux results in a much larger final recoil velocity.

Astrophysical Implications

We can also calculate the linear momentum flux for binaries with arbitrary masses and spin orientations, using the effective-one-body (EOB) model. This model includes an analytic description of the inspiral phase, a short merger, and a superposition of ringdown modes of a Kerr BH. By varying the matching point between inspiral and ringdown, we can estimate the systematic errors generated with this method. Within these confidence limits (typically \( \epsilon \approx 0.1 \)), we find close agreement with previously reported results from numerical relativity.

Using a Monte Carlo implementation of the EOB model, we are able to sample a large volume of BH parameter space and estimate the distribution of recoil velocities (Schnittman & Buonanno 2007). For a range of mass ratios \( 1 < m_1/m_2 < 10 \), spin magnitudes \( \alpha_1, \alpha_2 < 0.5 \), and uniform random spin orientations, we find that the fraction of binaries have recoil velocities greater than 500 km/s, with \( f_{\mu\nu} = 0.97 \), have kicks greater than 1000 km/s. These velocities likely are capable of ejecting the final BH from its host galaxy (see, e.g. Baker et al. 2009, Campolattari et al. 2007). The sample to comparable-mass binaries with \( \mu_1/\mu_2 < 1 \), the typical kicks are even larger, with \( f_{\mu\nu} = 0.66 \) and \( f_{\mu\nu} = 0.66 \).

TOTAL INTEGRATED RECOIL VELOCITY

The total integrated recoil velocity along with the relative contributions from each of the leading flux terms. The three colored curves are used to give the recoil velocity in the black curve. When the current quadrupole moment \( \mu_\nu \) is small, there is a large recoil (left), during the ringdown phase, a large current quadrupole significantly changes the anti-kick (right), an effect that is also clearly evident in equal-mass spinning binaries (Buonanno et al. 2007).

REFERENCES

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