A Demonstration of the Magnetic Mirror Effect

RONALD J. ALLEN
University of Saskatchewan, Saskatoon, Saskatchewan, Canada
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A description is given of the magnetic mirror effect exhibited by charged particles spiralling into a converging magnetic field. The apparatus makes use of a commercial electron tube filled with an inert gas, originally designed for measuring the specific charge of the electron. The necessary field configuration is provided by an ordinary bar magnet. Measurements of pitch angle at various points of the particle trajectories were made from photographs of the spiral orbits. The magnetic field strength corresponding to these points was determined independently from extensive field measurements carried out near the end of the bar magnet. The ratio of field strengths at two different points of the particle trajectory as a function of the spiral pitch angles at those points is compared with theory.

THE Electron TUBE

RECENTLY, a device for measuring the specific charge of an electron has become available.¹ This consists of a specially designed electron tube mounted between Helmholtz coils. The tube itself projects a well-collimated beam from a small hole in the anode. This beam is visible throughout its length, due to the presence of an inert gas in the tube envelope, although collision effects diminish the visibility as the path length increases. The beam can be deflected into conveniently small radii of the order of 0.75 cm by fields of approximately 50 G since the electrons have an energy of only about 65 eV. With the Helmholtz coils not energized the beam emerges vertically from the aperture in the anode, and it is found that by holding an ordinary bar magnet at an appropriate angle to the beam the electrons are forced into a helical path. As they spiral into a region of increasing field strength, the pitch angle of the helix increases until it turns back on itself; this is the mirror point. The tube is shown in Fig. 1, with the bar magnet above it. Photographs of typical trajectories are shown in Fig. 2(a) and (b); the mirror effect is clearly apparent. The photographs were taken with a tripod-mounted Graphic camera and close-up lens attachment.

We are interested in the path of a charged particle for which B varies along the trajectory, but is constant in time at every point. The motion is usually treated² in terms of a drifting center of

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¹ The author is a student in the fourth year of the Honours Physics course at the University of Saskatchewan.
gyration $Q$ and the radius of gyration $a$ as shown in Fig. 3. The pitch angle is $\theta$, measured between the velocity $v$ and the magnetic field direction at that point. For motion in this type of field, the magnetic moment of a noncolliding charged particle remains constant to a high order of approximation. As a consequence, the magnetic field strength and pitch angle at any two points of the trajectory obey the relationship

$$B_2/B_1 = \sin^2\theta_2/\sin^2\theta_1.$$ 

Let $B_2$ and $\theta_2$ correspond to initial values, and suppose the particle is spiralling into a region of increasing field strength. Then when the ratio reaches $\sin^2\theta_2/1$ the particle has energy only in a direction perpendicular to the field lines. This point in space is known as the mirror point, and the particle is then reflected back into the region of lower field.

Fig. 4. Magnetic field strength $B$ as a function of perpendicular distance $a$ from the symmetry axis of the field.

Fig. 5. Magnetic field strength $B$ as a function of distance $Z$ from the end of the magnet (parallel to the symmetry axis).

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**MEASUREMENTS**

With reference to Fig. 1 the bar magnet is shown clamped to a laboratory stand and oriented at an angle to the vertical. The meter stick was photographed along with the spirals to provide a simple method of scaling. The two tubular lamps on each side of the tube provided illumination for the meter stick and bar magnet, respectively. Filament power for the tube was supplied by a battery eliminator, and the anode voltage with a Heathkit PS-3 power supply. Plate voltage was set at about 65 V. This is not critical, and was freely adjusted to give a well-defined beam of suitable configuration to be photographed. Pictures were taken with time exposures in the order of five seconds at $f/11$, using Kodak Portrait Pan sheet film.

The field of the bar magnet was determined with a commercial Hall-effect gaussmeter for a fine grid of points near the end of the magnet, using a small pencil-shaped probe with a flat, sensitive surface 4 mm in diameter. By proper orientation of the probe for maximum reading the total field $B$ was measured (not just the axial component $B_z$). Sample graphs of $B$ are shown in Figs. 4 and 5; many more graphs were drawn as working diagrams. From such extensive measurements of the field, the value of $B$ at any point on the trajectory could be determined. The coordinates $(a, z_1)$ of the spiral at the mirror point, and $(a, z_2)$ at other points were measured with vernier calipers for a number of photographs. Figure 6 shows a typical photograph used in ob-
Fig. 6. A typical photograph showing the method used to obtain results. The metric scale shows clearly on the right, with the spiral in the middle of the photograph and the magnet in the upper left corner. The coordinates and pitch angle of the spiral are drawn in for illustration.

The coordinates \((a_x, a_z)\) and the symmetry axis are shown. In this axially symmetric field, the spiral lies on the surface of a tube of magnetic flux; thus the point \(a_z\) was established by extrapolating the field lines on this surface in the plane of the photograph out to a distance \(z_2\) from the end of the magnet. The errors involved in this procedure are included in the estimated errors in finding the field strength at this point. With the coordinates known, the values \(B_1\) and \(B_2\) could be determined from the appropriate graphs. The pitch angle \(\theta_1\) (at the mirror point) is \(\pi/2\); \(\theta_2\) was measured with a protractor. It should be mentioned here that \(\theta_2\) is properly the angle between the velocity vector and the field direction at that point on the orbit, whereas measurements from the photographs yielded the angle between the velocity vector and the axis of symmetry of the field. It is easily shown that the true pitch angle \(\theta_2\) is related to the measured pitch angle \(\theta_2'\) by the relationship \(\cos \theta_2 = \cos a \cos \theta_2'\), where \(a\) is the angle between the field axis and the tangent to the field direction at the orbit. Figure 3 shows this angle. This correction was applied in calculating the pitch angle, although it was found that for all of the experimental points the correction was less than anticipated error in measuring \(\theta_1\).

The experimental values of \(B_1(\theta_2)/B_1\) together with the estimated errors are shown plotted in Fig. 7. The solid curve represents the \(\sin \theta_2\) dependence predicted by the theory. Within the errors of the measurements, the theory agrees with the experimental points. Measurements for angles near zero and ninety degrees could not be accurately obtained from the photographs.

In addition to providing a quantitative test of the theory, the apparatus can also be used to illustrate the variations in the spiral shape with the initial pitch angle of particle entry into the field. Beginning with the magnet horizontal and the pole face adjacent to the tube envelope, the beam describes a semicircular orbit from the aperture back to the plate. As the angle between magnet and the horizontal is increased from zero, the beam shows one loop of a large spiral which includes the mirror point. Eventually two or three loops can be obtained, and with the magnet almost vertically above the tube a long thin spiral is formed. The mirror point finally moves out of the tube and the beam strikes the glass envelope. With a certain orientation of the magnet the spiral shape and size may be altered by adjusting the plate voltage.

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