

Correlation of Photon and Neutrino Fluxes in Blazars and Gamma Ray Bursts

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ABSTRACT

Relativistic black-hole jet sources are leading candidates for high energy (\gg TeV) neutrino production. The relations defining (a) efficient photopion losses of cosmic-ray protons on target photons and (b) $\gamma\gamma$ opacity of γ rays through that same target photon field imply clear multiwavelength predictions for when and at what energies blazars and GRBs should be most neutrino bright and γ -ray dim. The use of multiwavelength observations to test the standard relativistic jet model for these source is illustrated.

Subject headings: gamma-rays: bursts—neutrinos—theory

1. INTRODUCTION

Multiwavelength observations provide important information about radiation processes and properties of astrophysical sources that cannot be obtained by observations in a narrow waveband. The opening of the high-energy neutrino window (for review, see Learned & Mannheim 2000) will provide a new channel of information that, in conjunction with photon observations, will test models of these sources. The km-scale IceCube (Ahrens et al. 2004) reaches a total exposure of ~ 1 km³-yr in 2009 and its design sensitivity in 2011. Plans are also being made for a northern hemisphere KM3NeT neutrino telescope in the Mediterranean Sea.¹

Because of their rapid variability, large luminosity, and relativistic outflows, gamma-ray bursts (GRBs) and blazars have been considered as the most likely sources of ultra-high energy cosmic rays and neutrinos (Waxman & Bahcall 1997; Vietri 1998; Rachen & Mészáros

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1998; Alvarez-Muñiz et al. 2000). The expected low neutrino-induced muon event rate, even for the brightest γ -ray blazars (Atoyan & Dermer 2001; Neronov & Semikoz 2002) and GRBs (Dermer & Atoyan 2003; Guetta et al. 2004), and the increasing importance of the cosmic-ray induced neutrino background at lower energies (Karle et al. 2003; Gaisser et al. 1995), means that event detection is greatly improved by choosing appropriate time windows during periods of highest neutrino luminosity. The important time windows for neutrino detection from blazars might be thought to be during γ -ray flaring states and, for GRBs, around the t_{90} time or during some hours around the burst trigger, as suggested by the extended 100 MeV – GeV emission for GRB 940217 (Hurley et al. 1994) and delayed anomalous γ -ray emission components in GRB 941017 and a few other GRBs (González et al. 2003; González 2005), because that is when energetic particle acceleration is most vigorous. But photopion production is enhanced in conditions of high internal photon target density, so that times of most favorable neutrino detection could also be argued to take place during periods of low γ -ray flux as a result of attenuation by the dense internal photon gas.

Here we explore this issue, considering how to use multiwavelength observations (at optical, X-ray, and γ -ray energies for blazars, and at X-ray and γ -ray energies for GRBs) to define the most favorable conditions for efficient neutrino production. Violation of these predictions will call into questions the underlying assumptions currently used in models of GRBs and blazars.

2. Analysis

Blazars and GRBs are widely thought to be black-hole jet sources powered ultimately by accretion onto a black hole, or by the spin energy of the black hole. In both cases, observations show that collimated outflows of highly relativistic plasma are ejected by processes taking place from compact regions. In the internal shock model (Mészáros 2006; Piran 2005), collisions between faster and slower shells dissipate directed kinetic energy in the form of field energy and accelerated particles that radiate. After the collision, the energized shocked fluid shell expands on the light-crossing time scale r'_b/c or longer, where r'_b is the characteristic size of the radiating fluid element in the comoving frame, assumed spherical and isotropic in the comoving frame. The causality constraint implies that the size scale of the emitting region $r'_b \lesssim c\delta_D t_{var}/(1+z)$, where the measured variability timescale $t_{var} = 10^\tau t_\tau$ s, and δ_D is the Doppler factor.

Within this geometry, the relation between the measured νF_ν flux f_ϵ and the target photon emissivity $j'(\epsilon', \Omega') = d\mathcal{E}'/dV'dt'd\Omega'd\epsilon'$ is $f_\epsilon \cong \delta_D^4 V'_b \epsilon' j'(\epsilon', \Omega')/d_L^2$, where $\epsilon \equiv h\nu/m_e c^2$, primes refer to comoving quantities, and unprimed quantities to measured values. The lumi-

nosity distance $d_L = 10^\ell d_\ell$ cm, can be calculated for the standard Λ CDM universe ($h = 0.72$, $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$) for a source at redshift z , $\epsilon' j'(\epsilon', \Omega') \cong cu'_{\epsilon'}/4\pi r'_b$ for radiation emitted isotropically in the comoving frame, and $u'_{\epsilon'} = m_e c^2 \epsilon'^2 n'(\epsilon')$ is the spectral energy density of the radiation field.

We write the νF_ν flux as $f_\epsilon = f_{\epsilon_{pk}} S(x)$, where $S(x)$ is a spectral function of the variable $x = \epsilon'/\epsilon'_{pk} = \epsilon/\epsilon_{pk}$. Here $\epsilon_{pk} \equiv 10^j$ is the measured photon energy (in units of $m_e c^2$) of the peak of the νF_ν spectrum with peak flux $f_{\epsilon_{pk}} = 10^n f_\eta$ ergs $\text{cm}^{-2} \text{s}^{-1}$. Thus $\epsilon' n'(\epsilon') \cong 3d_L^2 f_{\epsilon_{pk}} S(x)/(cr'_b{}^2 \delta_D^4 m_e c^2 \epsilon')$. The rate at which protons lose energy through photohadronic processes is $t'_{\phi\pi}{}^{-1} \cong \epsilon' n'(\epsilon') \hat{\sigma} c$, where $\hat{\sigma} \cong 70 \mu\text{b}$ is the product of the γp photohadronic cross section and inelasticity (Atoyan & Dermer 2003), and the threshold condition $2\gamma'_p \epsilon' \gtrsim \epsilon'_{thr} \cong 400$ relates the proper frame proton Lorentz factor γ'_p and the internal photon energy.

The target comoving photon spectral energy distribution (SED) from quasi-isotropic emissions, whether it is the synchrotron and synchrotron self-Compton (SSC) fields or a cascade radiation field, is approximated as a broken power law $S(x) = x^a H(1-x) + x^b H(x-1)$ with νF_ν indices a and b (see Fig. 1; more general spectral forms can easily be treated). Here the Heaviside function $H(x) = 1$ for $x > 0$ and $H(x) = 0$ for $x < 0$. The photopion energy-loss rate of ultrarelativistic protons with Lorentz factor γ'_p interacting with photons with energy ϵ'_{pk} near the peak of the νF_ν spectrum is, from the proceeding considerations,

$$\rho_{\phi\pi} = \frac{3\hat{\sigma} d_L^2 f_{\epsilon_{pk}} (1+z)}{m_e c^4 \delta_D^5 t_{var}^2 \epsilon_{pk}}. \quad (1)$$

For the model target photon spectrum with $0 < a < 3$, $b < 0$,

$$t'_{\phi\pi}{}^{-1}(\gamma'_p) \cong \rho_{\phi\pi} \begin{cases} 2y^{b-1}/[(1-b)(3-b)], & y \gg 1 \\ 2y^{a-1}/[(1-a)(3-a)], & y \ll 1, 0 < a \lesssim 1 \\ (a-b)/[(a-1)(1-b)], & y \ll 1, 1 \lesssim a < 3 \end{cases} \quad (2)$$

(Dermer 2007), where $y \equiv \epsilon'_{thr}/2\gamma'_p \epsilon'_{pk} \cong \delta_D^2 \epsilon'_{thr}/2\gamma_p(1+z)\epsilon_{pk}$, and the Lorentz factor $\gamma_p = E_p/m_p c^2$ of an escaping proton as measured by a local observer is $\gamma_p \cong \delta_D \gamma'_p$. The condition $y = 1$ for protons with energy $E_p = E_p^{\phi\pi}$ interacting with photons with energy ϵ_{pk} implies that

$$E_p^{\phi\pi} \cong \frac{m_p c^2 \delta_D^2 \epsilon'_{thr}}{2(1+z)\epsilon_{pk}} \cong \frac{1.9 \times 10^{14} \delta_D^2}{(1+z)\epsilon_{pk}(\text{keV})} \text{ eV}. \quad (3)$$

The radiating fluid element will expand explosively following its rapid energization through shell collisions, or through external shocks formed when the outflow sweeps through the surrounding medium. Photopion processes can be certain to be efficient—assuming of course that ultrarelativistic protons are accelerated in black-hole jets—if the photopion energy-loss rate $\rho_{\phi\pi}$, eq. (1), is greater than the inverse of the light travel timescale,

$(1+z)/\delta_D t_{var}$. An energetic cosmic ray will therefore lose a large fraction of its energy into electromagnetic and neutrino radiations through photopion production when the jet Doppler factor

$$\delta_D < \delta_{\phi\pi} \equiv \left(\frac{3\hat{\sigma} d_L^2 f_{\epsilon_{pk}}}{m_e c^4 t_{var} \epsilon_{pk}} \right)^{1/4} = 10^{-10.64+(2\ell+\eta-\tau-j)/4} \cong 7.3 d_{28}^{1/2} \left(\frac{f_{-10}}{t_0 \epsilon_{pk}} \right)^{1/4}. \quad (4)$$

The same radiation field that functions as a target for photomeson processes is a source of $\gamma\gamma$ opacity. The photoabsorption optical depth for a γ -ray photon with energy ϵ_1 in a quasi-isotropic radiation field with spectral photon density $n'(\epsilon')$ is $\tau_{\gamma\gamma}(\epsilon_1) \cong r'_b \int_0^\infty d\epsilon' \sigma_{\gamma\gamma}(\epsilon', \epsilon_1) n'(\epsilon')$. Using a δ -function approximation $\sigma_{\gamma\gamma}(\epsilon', \epsilon_1) \cong \sigma_T \epsilon' \delta(\epsilon' - 2/\epsilon_1)/3$ for the $\gamma\gamma$ pair production cross section $\sigma_{\gamma\gamma}$ (Zdziarski & Lightman 1985) gives

$$\tau_{\gamma\gamma}(\epsilon_1) \cong \tau_{\gamma\gamma}^{pk} \left[\left(\frac{\epsilon_1}{\epsilon_1^{pk}} \right)^{1-b} H(\epsilon_1^{pk} - \epsilon_1) + \left(\frac{\epsilon_1}{\epsilon_1^{pk}} \right)^{1-a} H(\epsilon_1 - \epsilon_1^{pk}) \right] \quad (5)$$

(Dermer 2005) (see also Lithwick & Sari 2001; Barwick et al. 2006), where

$$\tau_{\gamma\gamma}^{pk} = \frac{\sigma_T d_L^2 f_{\epsilon_{pk}}}{4 m_e c^4 t_v \delta_D^4 \epsilon_{pk}} \quad (6)$$

and

$$\epsilon_1^{pk} = \frac{2\delta_D^2}{(1+z)^2 \epsilon_{pk}}. \quad (7)$$

Fig. 1 compares the δ -function approximation given by the above equations with accurate calculations using the results of Gould & Schröder (1967) (see Brown et al. 1973, for corrections).

At the Doppler factor $\delta_D = \delta_{\phi\pi}$ that allows for efficient photopion production, the $\gamma\gamma$ optical depth at photon energy ϵ_1^{pk} is, after substituting eq. (4) into eq. (6),

$$\tau_{\gamma\gamma}^{\phi\pi} = \frac{\sigma_T}{12\hat{\sigma}} \cong 800. \quad (8)$$

Whenever photopion production is important, γ rays with energies given by eq. (7) have to be highly extinguished by $\gamma\gamma$ processes when interacting with peak target photons with energy $\sim \epsilon_{pk}$, making it impossible to detect γ rays at these energies. This energy is

$$E_\gamma^{\gamma\gamma} = \frac{2m_e c^2 \delta_{\phi\pi}^2}{(1+z)^2 \epsilon_{pk}} = \frac{2m_e c^2 d_L}{(1+z)^2 \epsilon_{pk}^{3/2}} \sqrt{\frac{3\hat{\sigma} f_{\epsilon_{pk}}}{m_e c^4 t_{var}}} \cong \frac{10^{-24.26+\ell+(\eta-\tau-3j)/2} \text{ GeV}}{(1+z)^2} \cong \frac{0.055 d_{28} f_{-10}^{1/2}}{(1+z)^2 t_0^{1/2} \epsilon_{pk}^{3/2}} \text{ GeV}. \quad (9)$$

The energy of protons that interact most strongly with peak target photons through photopion processes under conditions when photopion processes must be important is, from eq. (3),

$$E_p^{\phi\pi} = \frac{m_p c^2 \delta_{\phi\pi}^2 \epsilon'_{thr}}{2(1+z)\epsilon_{pk}} \cong 10^{-10+\ell+(\eta-\tau-3j)/2} \text{ eV} \cong 1.0 \times 10^{13} d_{28} \sqrt{\frac{f_{-10}}{t_0 \epsilon_{pk}^3}} \text{ eV} . \quad (10)$$

3. Results

Table 1 lists the important quantities derived in this paper. The quantity $\delta_{\phi\pi}$ is the jet Doppler factor where photopion losses are guaranteed to be important for protons of escaping energy $E_p^{\phi\pi}$. Protons with this energy undergo photopion interactions primarily with peak target photons with energy $\sim \epsilon_{pk}$. $E_{\gamma}^{\gamma\gamma}$ is the energy of γ rays that are attenuated through $\gamma\gamma$ pair production primarily by peak target photons.

Consider the target photon variability time for the following source classes: flat spectrum radio quasars (FSRQs) and GRBs, both known sources of GeV radiation, and X-ray selected BL Lac objects (XBLs), of which over a dozen are known TeV sources. We define the variability time scale t_{var} as the measured time over which the absolute flux varies by a factor of 2; if a source varies by $N\%$ over time Δt , then $t_{var} = 100\Delta t/N$, keeping in mind that this is a conservative assumption for temporal variability given that quiescent or unrelated emissions can add a separate slowly or nonvarying background. R-band optical and RXTE PCA ($\approx 2 - 60$ keV) observations of 3C 279 and PKS 0528+134 show that day timescale optical and X-ray variability can be expected for FSRQs (Hartman et al. 2001; Mukherjee et al. 1999). Day timescale optical/UV and X-ray variability is also observed for TeV BL Lac objects (Pian et al. 1997; Błażejowski et al. 2005).

For canonical FSRQ values taken from observations of 3C 279 or PKS 0528+134, Table 1 shows that photopion production is already important at Doppler factors of $\sim 9 - 16$ during times of day-scale optical flaring, and these optical photons effectively extinguish all γ rays with energies $\gtrsim E_{\gamma}^{\gamma\gamma}/800^{1/(1-b)}$ (Fig. 1), which would certainly include all $\gtrsim 100$ GeV – TeV photons. Unfortunately, TeV telescopes have not so far been successful in detecting FSRQs, but monitoring of an FSRQ during an optical flare with low-energy threshold air Cherenkov telescope such as MAGIC would identify periods of likely neutrino emission. Photohadronic neutrino secondaries have energies $E_{\nu} \approx E_p^{\phi\pi}/20$ and so would be produced at $\sim 10^{17} - 10^{18}$ eV, providing possible sources for ANITA (Barwick et al. 2006), though outside IceCube’s optimal energy range.

For guaranteed importance of photohadronic production implied by day-scale X-ray

variability, the Doppler factors of FSRQs and TeV BL Lac objects like Mrk 421 have to be unexpectedly small, $\lesssim 3$. If the X-ray flaring timescale of FSRQs were hourly rather than daily, then $\delta_{\phi\pi}$ would more nearly correspond to Doppler factors $\sim 5 - 10$ inferred from unification studies and superluminal motion observations of blazars (Vermeulen & Cohen 1994; Urry & Padovani 1995). During episodes of highly variable X-ray flux, such sources should be invisible to GLAST, and \gg TeV neutrinos should be created if FSRQs are sources of ultra-high energy cosmic rays. For the XBL estimate, a 15 minute X-ray flaring timescale has already been assumed. Thus FSRQs are more likely than BL Lac objects to be high-energy neutrino sources for IceCube, which also follows if, as is likely, the external radiation field plays a strong role in neutrino production (Bednarek & Protheroe 1999; Atoyan & Dermer 2001, 2003).

The outcome of this analysis to identify periods of high-energy neutrino production is best for bright GRBs with peak fluxes of $\approx 10^{-6}$ ergs cm^{-2} s^{-1} and peak photon energy in the range 50 keV – 0.5 MeV that show $\lesssim 1$ s spikes of emission. The bulk factors, ≈ 100 , are consistent with widely considered outflow speeds in GRBs (see, e.g., Razzaque et al. 2004, who also treat $\gamma\gamma$ attenuation in GRBs). Perhaps 100 MeV photons could be observed, but the GLAST LAT should see no \gtrsim GeV photons if $\delta_D \lesssim \delta_{\phi\pi}$, which is the most favorable time for detecting 100 TeV – PeV neutrinos and is at an optimal energy for detection with km-scale neutrino telescopes. Bright X-ray flares with durations $\sim 10^2$ s observed hundreds to thousands of seconds after the GRB trigger, like those discovered with Swift (O’Brien et al. 2006; Burrows et al. 2007; Murase & Nagataki 2006), with blast wave Doppler factors ≈ 50 , are also promising times to look for neutrinos and a γ -ray spectrum attenuated above ~ 100 GeV γ rays.

The condition $\tau_{\gamma\gamma}(\epsilon_1) < 1$ for detected γ rays with energy ϵ_1 gives a minimum Doppler factor

$$\delta_D^{min} = \left[\frac{\sigma_T d_L^2 f_{\epsilon_{pk}}}{4m_e c^4 t_v \epsilon_{pk}^A} \left(\frac{(1+z)^2 \epsilon_1}{2} \right)^{1-A} \right]^{1/(6-2A)} \quad (11)$$

(Dermer 2005), with $A = b$ if $\epsilon_1 < \epsilon_1^{pk}$, and $A = a$ if $\epsilon_1 > \epsilon_1^{pk}$. If $\delta_D^{min} \gtrsim \delta_{\phi\pi}$, we should not expect GRBs to be neutrino bright. Synchro-Compton analysis of high-quality radio and gamma-ray blazar SED data with resolved VLBI cores and self-absorption frequencies give the Doppler factor directly, provided the synchrotron self-Compton component can be identified and separated from external Compton and photohadronic emission components. Should high-energy neutrinos be detected when the Doppler factor inferred from these tests is greater than $\delta_{\phi\pi}$, then this would call into question our understanding of the structure of black-hole jets, for example, the assumption of isotropy of target photon distributions in the comoving jet frame.

4. Summary and Conclusions

We have presented a detailed treatment of combined photomeson and gamma-ray opacity, which is a crucial problem that unites the neutrino particle physics and the electromagnetic worlds. This problem has been numerically treated for GRBs (Dermer & Atoyan 2003), but no detailed analytical treatment is present in the scientific literature. GLAST observations will reveal if γ -ray spectra of FSRQ blazars and GRBs show evidence for strong γ -ray absorption during periods of variable target photon emissions, signifying favorable conditions for high-energy neutrino production.

To illustrate the use of these results, suppose that a blazar or GRB with measured redshift z is monitored at optical, X-ray, or soft γ -ray energies, giving a light curve $f_\epsilon(t)$ at photon energy $\epsilon(t)$. The structure of the light curve implies $t_{var}(t)$. From these observables, the Doppler factor $\delta_{\phi\pi}(t)$ for guaranteed importance of photomeson production is derived from eq. (4). If high-energy neutrinos are detected, then the source must be opaque at γ -ray energies given by eq. (9). If γ rays are detected at these energies, then the basic relativistic jet model must be wrong. Suppose instead that γ rays at some energy ϵ_1 are detected. In this case, the minimum Doppler factor $\delta_D^{min}(t)$ can be inferred from eq. (11) to define times when these sources can and cannot be neutrino bright.

Times and locations of bright, variable MeV γ -ray and extincted GeV fluxes from GRBs can be done exclusively with the GLAST GBM and LAT, whereas other tests for γ -ray/multiwavelength correlations giving the most favorable times for high-energy neutrino detection in blazars require collaboration and coordination of separate facilities. The necessary organization is already underway between GLAST and the ground-based γ -ray telescopes, e.g., HESS and VERITAS, but blazar observations with, e.g., Swift, Suzaku, and RXTE correlated with GLAST, AGILE, and ground-based high-energy γ -ray telescopes will be crucial for neutrino discovery science and testing models of relativistic jet sources.

We thank Armen Atoyan for discussions. The work of CD is supported by the Office of Naval Research and a GLAST Interdisciplinary Scientist Grant. TL's research is supported by the GLAST grant and a Swift Guest Investigator Grant.

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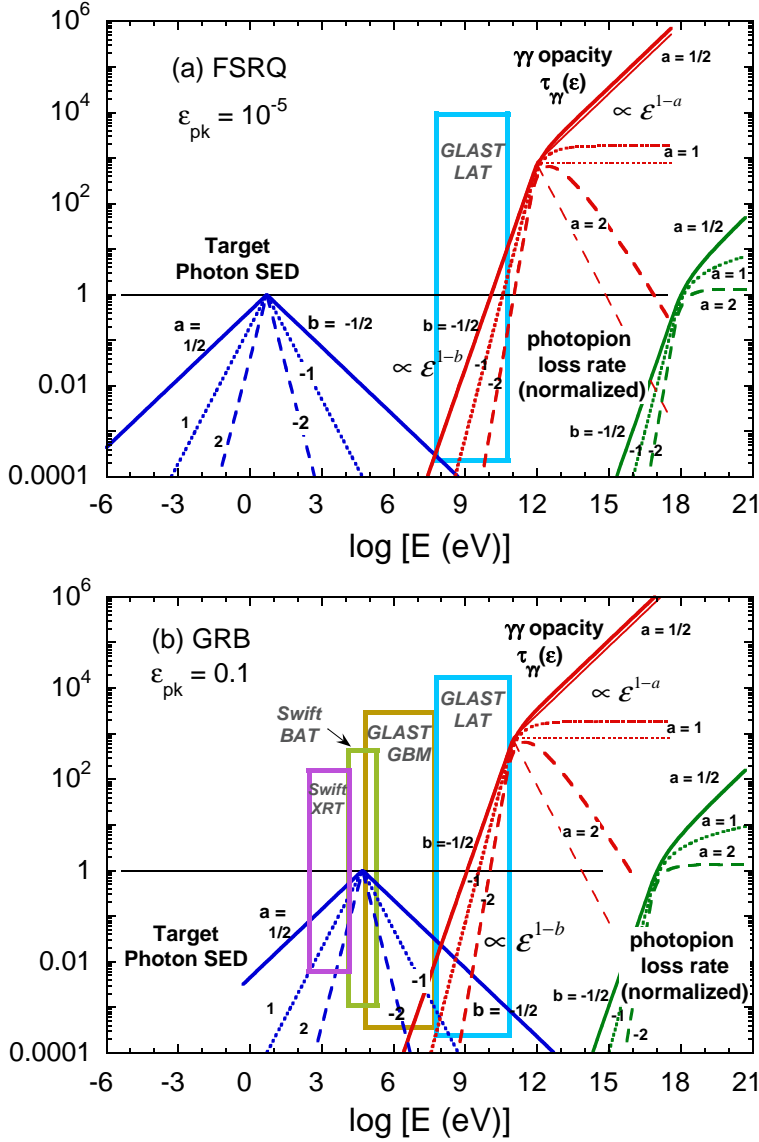


Fig. 1.— Target photon SED, $\gamma\gamma$ opacity $\tau_{\gamma\gamma}(\epsilon_1)$, and normalized photopion energy-loss rate $t'_{\phi\pi}{}^{-1}/\rho_{\phi\pi}$ are shown as a function of photon or neutrino energy for parameters of (a) optically flaring FSRQ and (b) rapidly varying prompt GRB emission, using parameters from Table 1. The target photon SED is approximated as a broken power law with νF_ν flux peaking at ϵ_{pk} . When photopion processes are certain to be important for protons with energy $E_p^{\phi\pi}$ that interact with peak target photons with energy $\approx \epsilon_{pk}$, then the $\gamma\gamma$ opacity of γ rays with energy $E_\gamma^{\gamma\gamma}$ is ≈ 800 . The $\gamma\gamma$ opacity is less than unity at photon energies $\lesssim E_\gamma^{\gamma\gamma}/800^{1/(1-b)}$. Heavy and light curves for $\tau_{\gamma\gamma}(\epsilon_1)$ are accurate numerical integrations (Gould & Schröder 1967) and δ -function approximations (Dermer 2005), respectively.

Table 1. Doppler Factor $\delta_{\phi\pi}$, γ -Ray Photon Energy $E_{\gamma}^{\gamma\gamma}$, and Cosmic Ray Energy $E_p^{\phi\pi}$

	ℓ^a	η	τ	j	$\delta_{\phi\pi}$	$E_{\gamma}^{\gamma\gamma}(\text{GeV})$	$E_p^{\phi\pi}(\text{eV})$
FSRQ	28.7	-11	5	-5 (5 eV)	9	92	5×10^{17}
IR/optical				-6 (0.5 eV)	16	30×10^3	1.6×10^{19}
FSRQ	28.7	-11	5	-2 (5 keV)	1.6	0.03	1.6×10^{13}
X-ray				-3 (0.5 keV)	2.8	0.92	5×10^{14}
XBL	27	-10	3	-2 (5 keV)	1.3	0.14	3×10^{13}
X-ray				-3 (0.5 keV)	2.3	4.7	9×10^{14}
GRB	28.7	-6	0	0 (511 keV)	160	2.9	2×10^{15}
γ ray				-1 (51 keV)	280	92	5×10^{16}
X-ray flare		-9	2	-3 (0.5 keV)	50	290	1.6×10^{17}

^aSources at $z = 2$ except for XBLs, at $z \approx 0.08$, $d_L = 10^{27}$ cm.