



Accuracy of the HST Standard Astrometric Catalogs *w.r.t.* Gaia

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Abstract

The goal of astrometric calibration of the HST ACS/WFC and WFC3/UVIS imaging instruments is to provide a coordinate system free of distortion to the precision level of ~ 0.1 pixel (~ 4 -5 mas) or better. This astrometric calibration is based on two HST astrometric standard fields in the vicinity of the globular clusters, 47 Tuc and ω Cen, respectively. The derived calibration of the geometric distortion is assumed to be accurate down to 2-3 mas. Is this accuracy in agreement with the true value? Now, with the access to globally accurate positions from the first Gaia data release (DR1), we found that there are measurable offsets, rotation, scale and other deviations of distortion parameters in two HST standard astrometric catalogs. These deviations from the distortion-free and properly aligned coordinate system should be accounted and corrected for, so that the high precision HST positions are free of any systematic errors. We also found that the precision of the HST pixel coordinates is substantially better than the accuracy listed in the Gaia DR1. Therefore, in order to finalize the components of distortion in the HST standard catalogs, the next release of Gaia data is needed.

1. Introduction

The HST imaging instruments, ACS/WFC and WFC3/UVIS and IR, have a considerable optical field-angle distortion due to the optical assembly of the telescope. By design, the focal plane of both cameras is tilted with respect to the incoming beam by $\sim 28^\circ$ and

$\sim 21^\circ$, respectively. As a result the HST WFC3 and ACS/WFC images are distorted by up to $\sim 7\text{--}11\%$ across the detectors. This does not effect the image quality, but introduces a strong shift in positions that must be astrometrically calibrated and corrected prior to any scientific astronomical studies.

The first ACS/WFC geometric distortion calibrations by Anderson (2002) were based on a self-calibration technique using the ACS/WFC images acquired right after installation of the ACS camera in 2002. The high-order polynomial distortion was calibrated to high accuracy, but the plate-scale was less constrained. The systematic trend of the plate-scale in the original (2002) solution introduces a poor alignment of ACS/WFC drizzled images.

The astrometric reference field in the vicinity of ω Cen (Anderson & van der Marel, 2010) was used for the geometric calibration of the WFC3/UVIS and IR channel. This provided a high-accuracy characterization of both - the linear part and as well the high-order polynomial distortion (Kozhurina-Platais *et al.* 2009 a,b).

The successful WFC3 geometric distortion calibration was then used to improve the long-standing problem in the ACS/WFC geometric distortion calibration. As described by Kozhurina-Platais *et al.* (2009, 2015), the geometric distortions calibration of WFC3 and ACS/WFC are based on the standard astrometric catalogs in the vicinity of the globular clusters ω Cen and 47 Tuc, respectively. It is straightforward to solve for distortion, particularly for scale and rotation, using calibrated pixel coordinates from an astrometric standard field: the measured pixel position of stars in the coordinate system of the detector are compared with the position of the same stars in an astrometric standard catalog, which are supposedly distortions free. The residuals between the measured position of stars and the corresponding position in an astrometric standard catalog then directly reveal the geometric distortion: basic geometry of the optical projection, possible deviation in the focal length, optical aberrations, etc.

The precise positions in the standard astrometric catalog were instrumental in revealing the high-frequency systematics in the raw X&Y measured positions from ACS/WFC and WFC3/UVIS observations. In particular, the high positional precision of the standard catalog were crucial in the discovery of CCD pixel-grid irregularities due to the lithographic-mask pattern in the WFC3/UVIS detector (Kozhurina-Platais *et al.* 2010) later confirmed by Bellini *et al.*, (2011). Similarly to WFC3/UVIS, comparison of precise positions between the standard astrometric catalog and the ACS/WFC positions unveils two-dimensional (2-D) pixel-grid irregularities due to the lithographic-mask pattern in the ACS/WFC detector (Kozhurina-Platais *et al.* 2015), as opposed to a 1-D look-up-table of corrections along of every 68th columns in X direction only (Anderson, 2002).

On the other hand, an astrometric standard catalog may contain itself systematic errors and have an inaccurate alignment to the International Celestial Reference System (ICRS), which will propagate as an additional *catalog distortion*.

The significantly high levels of precision and accuracy of the ESA/Gaia space mission, provide new opportunity to test the accuracy of the HST/ACS astrometric catalogs. The first Data Release 1 (DR1) of the Gaia mission is an all-sky catalog of positions for 1.1 billion sources (Gaia wide band G_mag down to 20.7) from one year of observations (Gaia Collaboration *et al.*, 2016a; Gaia Collaboration *et al.* 2016b). Each source in the catalog has a large number of parameters that fully characterizes its positional accuracy at the exact epoch of 2015.0.

The aim of this report is three-fold. First, we provide a short description of the standard astrometric catalogs and its comparison with Gaia/DR1. Second, we give a detailed account of the astrometric errors found in the HST catalogs *w.r.t.* Gaia, assuming that the Gaia positions are perfectly accurate. Finally, we describe how the HST standard catalogs can be improved and corrected for and how that will affect the accuracy of HST ACS/WFC and WFC3/UVIS final geometric distortion correction.

2. HST ACS Standard Astrometric Catalogs

2.1. Gaia DR1 source positions in the vicinity of 47 Tuc

The astrometric reference frame is based on nearly all existing ACS/WFC observations of 47 Tuc from 2001 to 2006 through the F606W filter. These observations are characterized by wide range of offsets and orientations and were used to create a master list of stellar positions. The tangent-plane positions of stars in this master list are given for the average epoch of 2004.5 and are accurate to ~ 0.02 ACS/WFC pixel (1 mas) across of the entire catalog according to Anderson (2007). The reference point of this catalog is at $RA=00^h22^m35^s.0$ & $Dec=-72^\circ04'00''$ corresponding to pixel (5000,5000) at the master list positions. Each star in this catalog has its relative proper motion with respect to the bulk motion of 47 Tuc (see Kozhurina-Platais, *et al.*, 2015).

All Gaia sources in Data Release 1 from the Gaia Archive Core System located at European Space Agency (ESA) Center, were retrieved within 0.5° of the reference point. Figure 1 shows the spatial distribution of Gaia sources in the vicinity of 47 Tuc.

Since the majority of the ACS/WFC 47 Tuc observations were taken at about $6.7'$ South-West from the center of this globular cluster, there is a strong gradient in the stellar

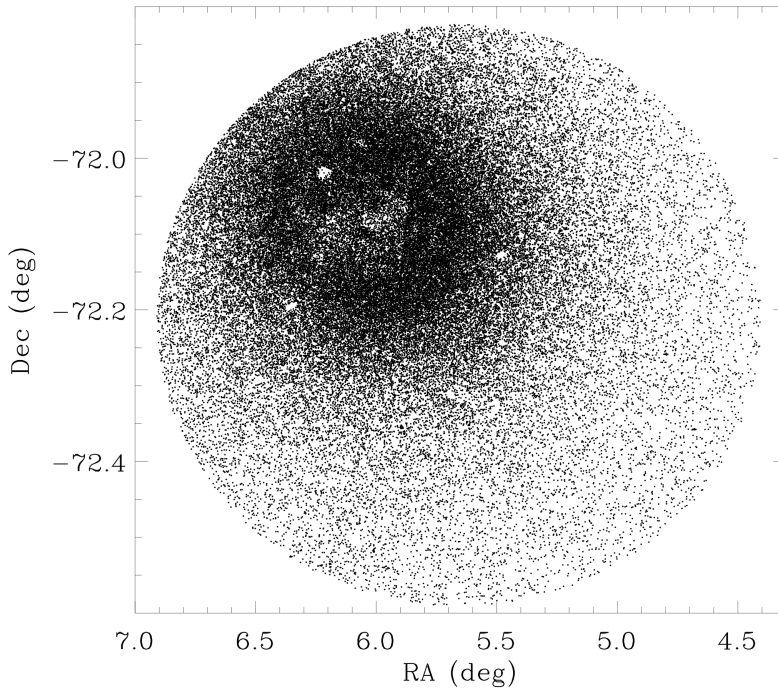


Fig. 1.— Spatial distribution of Gaia sources in the vicinity of 47 Tuc. The sky positions RA and Dec are in decimal degrees.

radial-density profile. As seen in Figure 1, stars are more concentrated toward the center of the cluster on the North-East. Despite this gradient, there is a sufficient number of stars (~ 4000) in all parts of the field to provide an accurate calibration of the ACS/WFC geometric distortion.

The Gaia DR1 positional errors were used to estimate the necessary cutoff for comparison between the HST ACS/WFC and Gaia/DR1. Figure 2 shows the positional errors in RA and Dec for all selected stars in the vicinity of 47 Tuc. The distribution of the errors is uneven due to the satellite scanning ‘law’, designed to achieve the best possible coverage of the sky¹. The distribution of the errors is also changing in the sets of downloaded Gaia images. On average, the positional errors in RA are two times larger than those in Dec .

As expected, the positional errors are increasing as function of stellar magnitudes. Figure 3 shows the error distribution in RA as function of Gaia’s G-magnitude. Another source of positional error is the image blending due to the crowding in the core of globular clusters

¹<http://sci.esa.int/gaia/58214-gaia-scanning-the-sky/>

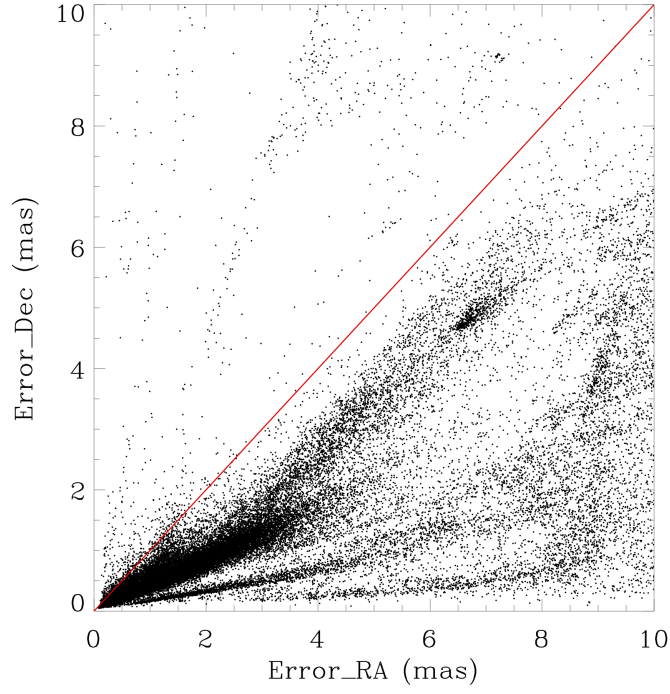


Fig. 2.— Positional errors of Gaia stars in the vicinity of 47 Tuc. The diagonal red line indicates an even distribution of the errors.

(Pancino, *et al.*, 2017). In order to keep high an astrometric accuracy of Gaia positions, only stars with positional errors $\lesssim 2$ mas were used in this analysis.

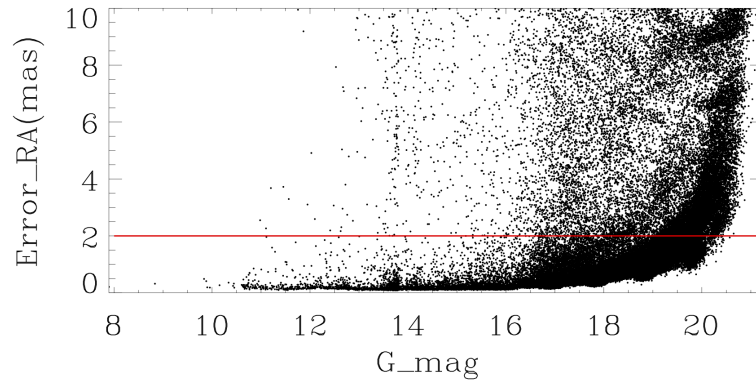


Fig. 3.— Positional errors of Gaia stars in *RA* as a function of G-band magnitude for selected stars in the vicinity of 47 Tuc. Only stars below the red line are selected for the analysis.

2.2. Gaia DR1 source positions in the vicinity of ω Cen globular cluster

The Gaia sources were also retrieved around the center of the globular cluster ω Cen, which serves as a standard astrometric catalog for the WFC3/UVIS and IR geometric distortion calibrations. Figure 4 shows the spatial distribution of Gaia sources in the vicinity of ω Cen.

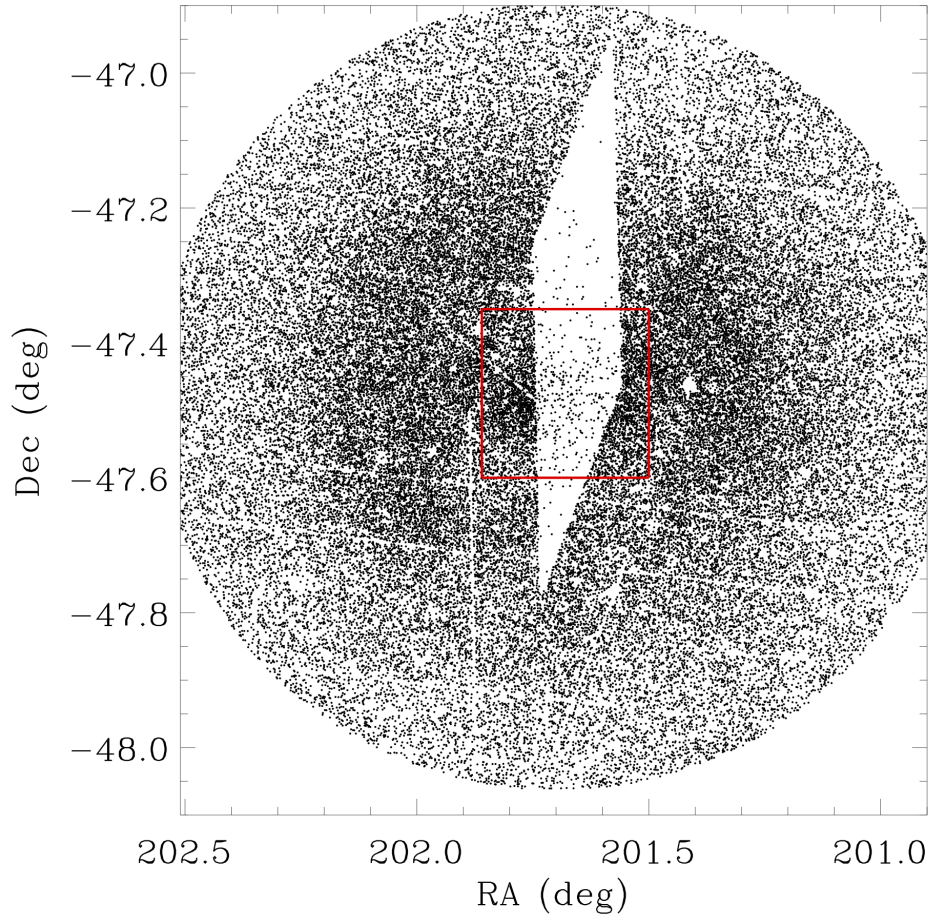


Fig. 4.— Spatial distribution of Gaia sources in the vicinity of the globular cluster ω Cen. The red box at the center of the cluster indicates the boundary of the HST standard astrometric frame. The parallelogram-shape in the center is due to on-board memory limitations of Gaia. The sky positions RA and Dec are in decimal degrees.

The spatial distribution of Gaia sources shown in Figure 4 is not uniform. The parallelogram-shape void near the center of ω Cen is the results of the Gaia DR1 limitation due to the extreme stellar density of the region and crowding issues in the core of the globular cluster

ω Cen. As discussed in Pancino *et al.* (2017), crowding errors will limit Gaia’s astrometric performance for the majority of dense stellar clusters. In addition, the Gaia catalog presents some visible traces (narrow strips) from left to right and from top to bottom, which is an indications of the prevalent directions of scanning. The location of standard catalog in the vicinity of ω Cen (Fig.4) is very poorly sampled in Gaia DR1, making the latter hardly suitable for comparison. On the optimistic side, significant improvements in observation of ω Cen are expected in the 2018 Gaia release. As for now, the analysis of standard astrometric catalog in the vicinity of ω Cen is put on hold for the future Gaia data releases.

3. Evaluation of the 47 Tuc standard catalog with Gaia

Gaia DR1 source positions are given in the ICRS equatorial system defined by Right Ascension (α) and its Declination (δ), while the positions of the HST standard astrometric catalog are given in the tangent-plane X&Y pixel system, ostensibly aligned along (α) and (δ).

In order to compare these two coordinates systems we took into account some practical considerations. First, we wish to maintain the accuracy and precision of the pixel coordinates. Second, the maximum precision is preserved only if all data manipulations are performed in the original pixel coordinates. And third, the transformation from one coordinate system into another may introduce model-dependent systematic, also known as modeling errors, depending on how complicated the transformation is. Because of that we chose to project the Gaia equatorial positions on the same tangent-plane as the HST standard catalog. The tangent-plane (*a.k.a.* gnomonic projection) was performed at a reference point of the HST catalog $\alpha=5.64583349^\circ$ and $\delta=-72.06666565^\circ$, corresponding to pixel (5000,5000) with the exact scale factor of 50 mas/pixel. Then, these $U\&V$ tangent-plane Gaia positions were cross-identified to the stars in the HST standard catalog positions, employing the linear transformations (shift, scale and rotation) based on iterative least-squares minimization. A poorly-measured, saturated stars and cosmic rays were interactively rejected, and the stellar positions in the HST standard catalog were simultaneously updated for proper motion of stars in 47 Tuc (Kozhurina-Platais, *et al.*, 2015) to the epoch of Gaia/DR1 equal to 2015.0.

3.1. Linear transformations between two coordinates systems

The evaluation of the HST catalog was done by employing the linear transformation of X,Y catalog positions into the system of the orthogonal reference coordinates U and V ,

which represent Gaia positions here. Then the linear terms in this transformation between two coordinate systems are used to detect possible residual systematic in the catalog positions and its characterization of an accuracy. The following linear transformation between two systems was used:

$$U = A_1 + A_2X + A_3Y \quad (1)$$

$$V = B_1 + B_2X + B_3Y \quad (2)$$

where U & V are the positions of the orthogonal coordinate system (tangent-plane Gaia positions), and X & Y are the HST standard catalog positions to be evaluated.

A general linear transformations from one system (X,Y) into another (U,V) have 3×2 parameters. The meaning of these parameters have been discussed in several articles (e.g., Eichhorn, 1957, Jefferys 1963, Eichhorn & Williams 1963, Taff 1980, Kiselev 1989), and define the basic geometry between two coordinates systems; the parameters A_1 and B_1 are the constant terms and represent an arbitrary offset between two sets of coordinates systems; parameters A_2 , A_3 , B_2 , B_3 are the linear terms characterizing the scale and rotation between two coordinates systems. The rotation angle between these two systems then can be defined as:

$$\tan(\theta) = \frac{A_3}{B_3} \quad (3)$$

The skew term, which is the total amount of non-orthogonality between the principal X & Y axes, can be derived from the following ratio of linear terms:

$$\tan(\gamma) = -\frac{A_2A_3 + B_2B_3}{A_2B_3 - B_2A_3} \quad (4)$$

The scale term is proportional to:

$$\mathcal{M} = \sqrt{(A_2B_3 + B_2A_3)} \quad (5)$$

These linear terms were re-parametrized by Anderson (2007) in the HST context, and were used by van der Marel *et al.* (2007) in the calibration of the ACS/WFC absolute

scale. These parameters were adopted by Sahlmann (2017) for the validation of the JWST calibration field catalog. To be consistent with these authors, the linear terms of Eqs.1-2 were parametrized here again in the HST context.

The off-axis skew, representing non-perpendicularity between the X , Y axes is defined as $Skew_{off-axis} = (A_2 - B_3)/2$. The on-axis skew, representing the scale difference between the X , Y axes is: $Skew_{on-axis} = -(A_3 + B_2)/2$.

The scale in X&Y axes are: $Scale_X = \sqrt{(A_2^2 + A_3^2)}$ and $Scale_Y = \sqrt{(B_2^2 + B_3^2)}$, respectively. The global scale then is $Scale = \sqrt{(A_2B_3 - A_3B_2)}$.

The rotation in X axis is $Rot_X = \arctan(-A_2/B_2)$ and that in Y-axis is $Rot_Y = \arctan(A_3/B_3)$. The global rotation is then $Rot = \arctan((A_3 - B_2)/(A_2 + B_3))$, and the deviation from non-perpendicularity between X&Y axes is global skew: $Skew = (Rot_X - Rot_Y)$.

Table 1 shows the numerical values of the derived parameters and their uncertainties as defined above. The parameters were calculated by least-squares minimization using 2312 common stars between the two catalogs, Gaia and HST. The solution provides an RMS of 0.97 mas in X and 0.78 mas in Y. The current Gaia precision is the main contribution to the estimated RMS.

Table 1: The values of the parameters and its standard errors from Eqs. 1–5

Parameter	Values	STD. Errors	Units
Global offset in X	129.45	0.65	mas
Global offset in Y	-717.80	0.55	mas
Global Scale	0.99991299	0.53e-07	–
Global Rotation	-0.00946781	0.31e-07	°
Global Skew	0.00052191	0.21e-07	°
Rotation in X	-0.00972876	0.41e-07	°
Rotation in Y	-0.00920685	0.41e-07	°
Scale in X	0.99991385	0.69e-07	–
Scale in Y	0.99991212	0.38e-07	–
Skew - on-axis	8.6500e-07	0.31e-08	–
Skew -off-axis	4.5543e-06	0.40e-07	–

The calculated parameters in Table 1 show a significant offset, rotation and scale difference between the two catalogs, and marginally significant skew terms.

Two-dimensional X&Y residuals maps were used below to visualize directly the contribution of each of the linear terms of transformation.

3.1.1. *Effects of X and Y shifts*

Figure 5 shows the positional differences between two coordinates systems, with the vectors indicating of the global offset dominated in Y-axis.

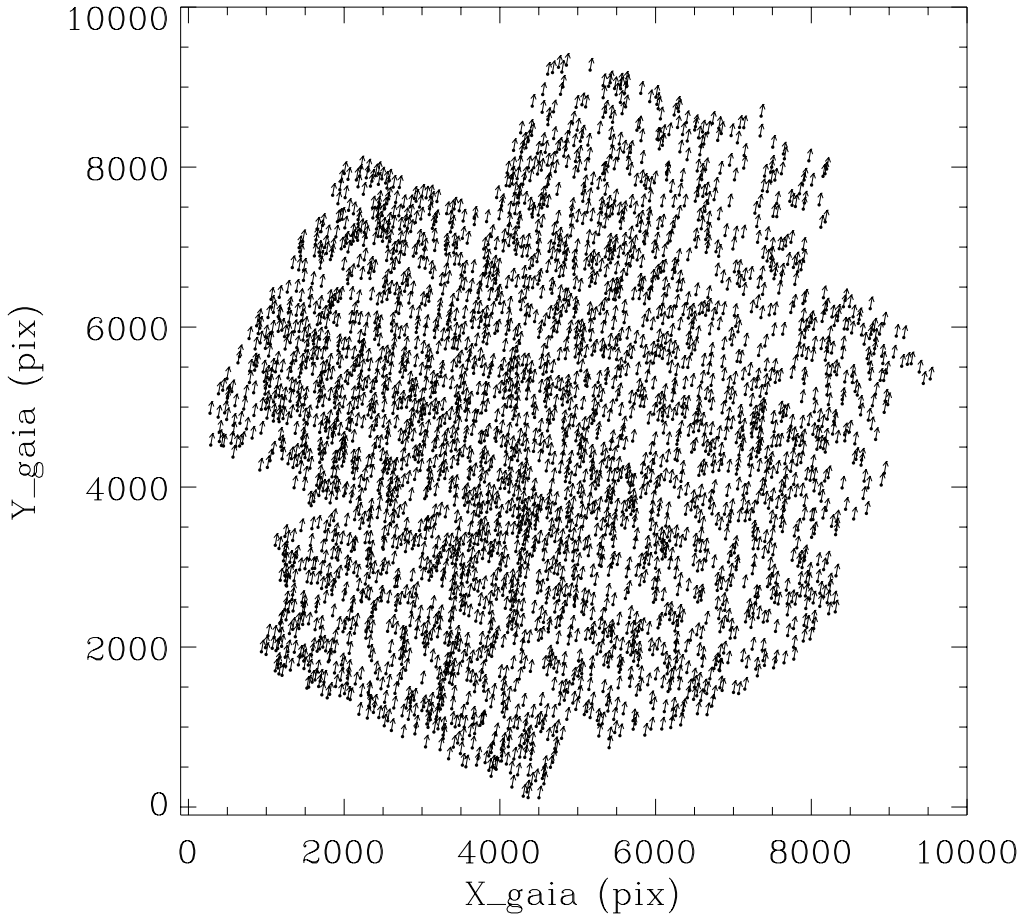


Fig. 5.— The difference in position between the Gaia and HST coordinates systems before applying global offsets in X and Y positions. Each vector magnified by 10. The tangent-plane positions X and Y are in units of the ACS/WFC pixels.

3.1.2. Effect of Rotation

After applying the linear shift in both coordinates, a significant solid-body type rotation between the two coordinate systems is clearly seeing. Figure 6 shows rotation of the HST catalog relative to the Gaia catalog. The global rotation between two catalogs, calculated from the linear parameters is equal to $-35.02 \pm 0''.10$ (Table 1).

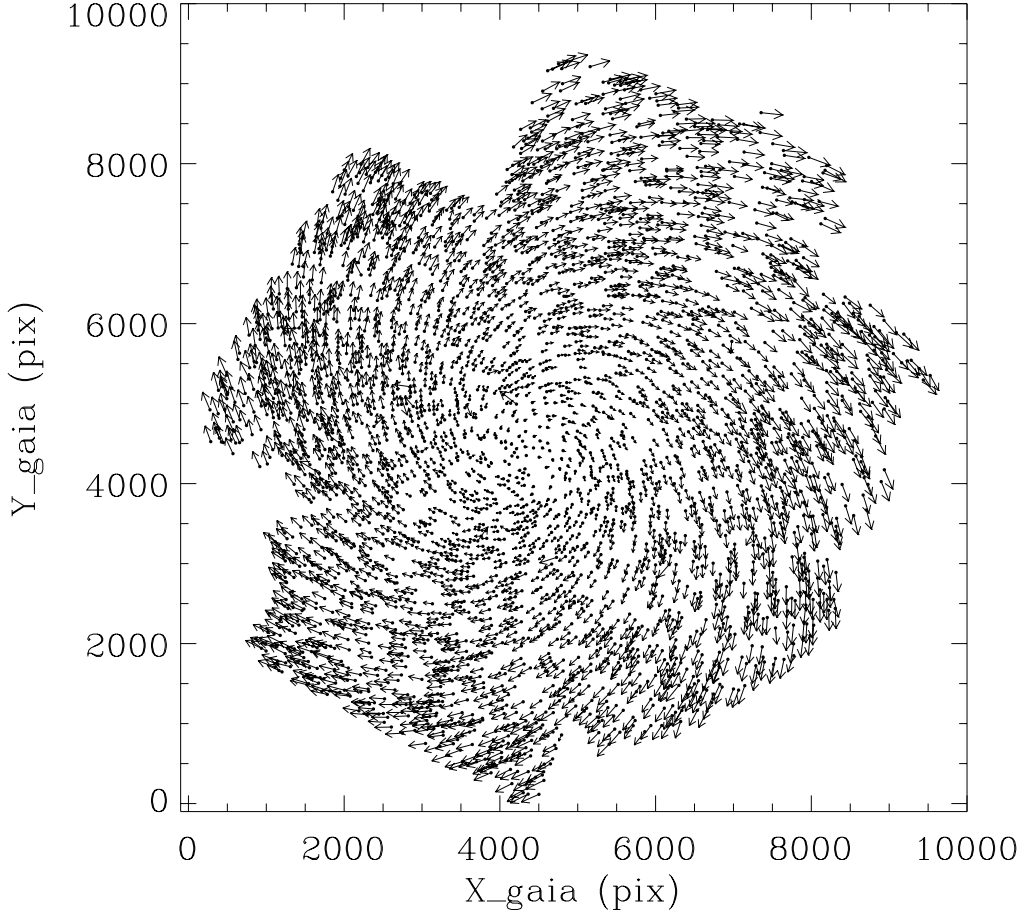


Fig. 6.— The effect of global rotation between the two coordinate systems after subtraction of global shifts in X and Y positions. The maximum length of the vector is ~ 1.3 pix (65 mas). Each vector is magnified by a factor 300. The tangent-plane positions X and Y are in the units of ACS/WFC pixels.

3.1.3. *Effect of Scale*

After applying both global shift and rotation, the pattern of X&Y residuals shows a vector growing pattern from the central point towards the edges and, varies with position over the field. According to Anderson (2007), this catalog was constructed from the set of ACS/WFC F606W images taken at different pointing, roll-angles, and over 5-year time period. The reference point corresponding to the center of the catalog located at $X=5000$ and $Y=5000$ pixels chosen as the tangential point for transformation of all sets of F606W images tangential X&Y positions to this reference point. As pointed out in van Altena (2013), the accuracy of transformations between two tangential-coordinate systems depends on how accurately the position of the optical center is known. The location of ACS/WFC in the HST focal-plane is off from the telescope optical center, thus the transformations between two tangential-plane systems are on the surface of a sphere. If the distance on the sphere between two respective tangential points is small, then a shift and rotation would be sufficient. However, the center of the ACS/WFC image is separated by $\sim 360''$ on arc from the HST optical center². This requires a gnomonic projection with the tangent point tied to the telescope’s optical center. If this is ignored, the plate scale will vary across the field of view. The global scale of the HST catalog is calculated as $\sqrt{(A_2B_3 - A_3B_2)}$, and is equal to 0.99991299 at the adopted reference point (see also Table 1). But, as seen in Figure 7, the scale is not constant over the field. The variation of the global scale from the center to the far edges of the standard catalog is of the level of 8×10^{-5} .

To calculate the difference in scale between the Gaia and the HST catalog, we took the ratio of $(1.0/0.999913)$, where the denominator is the global scale (Table 1) and multiplied by the adopted scale of 50 mas, will equal to the true scale. Thus, the true scale of the HST catalog is larger by 0.004 mas than previously assumed and that introduces 17 mas (0.35 pix) error at the edge (4000 pixel) of the catalog.

4. **Validation of the improved HST astrometric catalog in the vicinity of the globular cluster 47 Tuc.**

The derived transformation parameters from Eqs.1-2 in Sec.3.1 were used to compute inverse transformations to the positions of the HST catalog. These positions were then linearly transformed into the system of the Gaia reference frame, i.e. a corrected HST X&Y catalog of positions aligned to the Gaia reference frame. Figure 8 shows the difference

²<http://www.stsci.edu/hst/observatory/apertures>

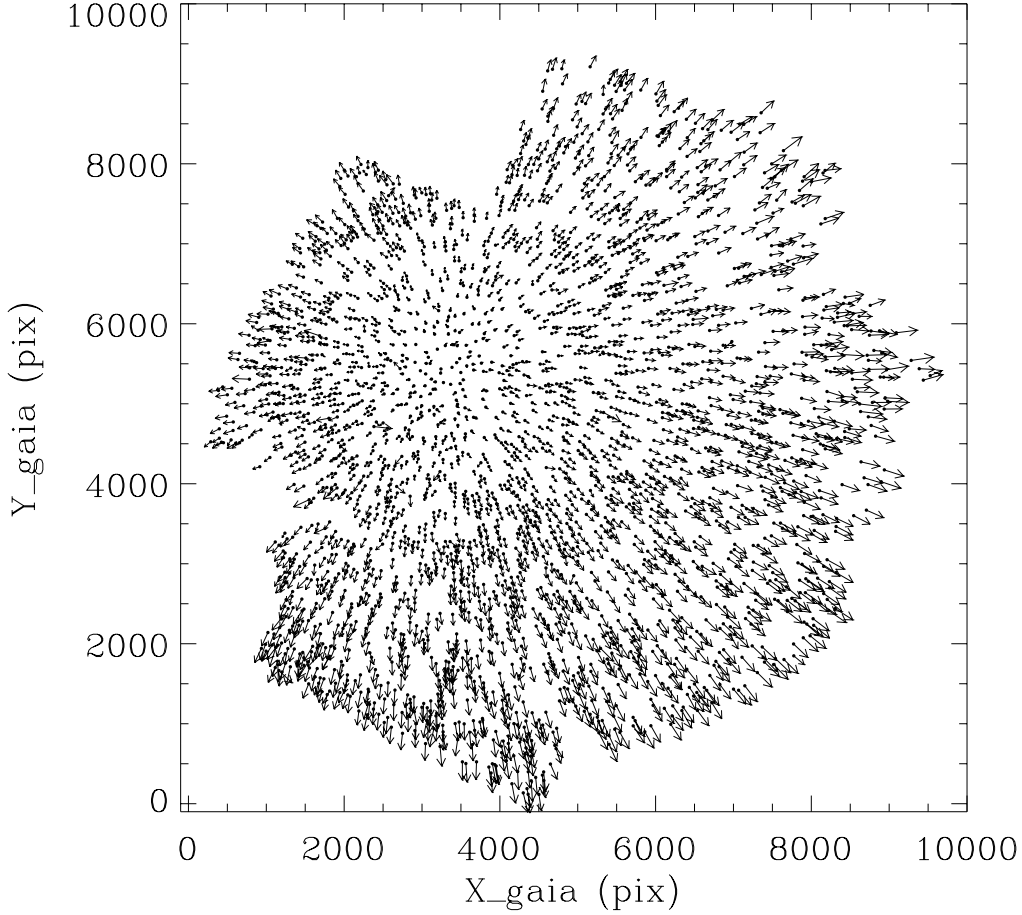


Fig. 7.— Difference between the Gaia and HST coordinate systems after subtraction of both global shift and rotation. There is a clear indication of a difference in scale. The size of the maximum vector is 0.7 pix (35 mas). Each vector is magnified by a factor of 300. The tangent-plane positions X and Y are in the units of the ACS/WFC pixels.

between the two coordinates systems after the correction of the HST standard catalog. Figure 8 shows that the large structural residual patterns seen in Figures 5, 6, 7 are now eliminated. The distribution of the residuals is random, with the largest accidental errors of ~ 0.3 pix. Thus, the Gaia positions provide rigid and exacting constraints on the linear terms of distortion: scale, rotation and skew. However, in terms of precision the Gaia DR1 is not yet adequate for the final assessment of the accuracy of the HST standard astrometric catalog.

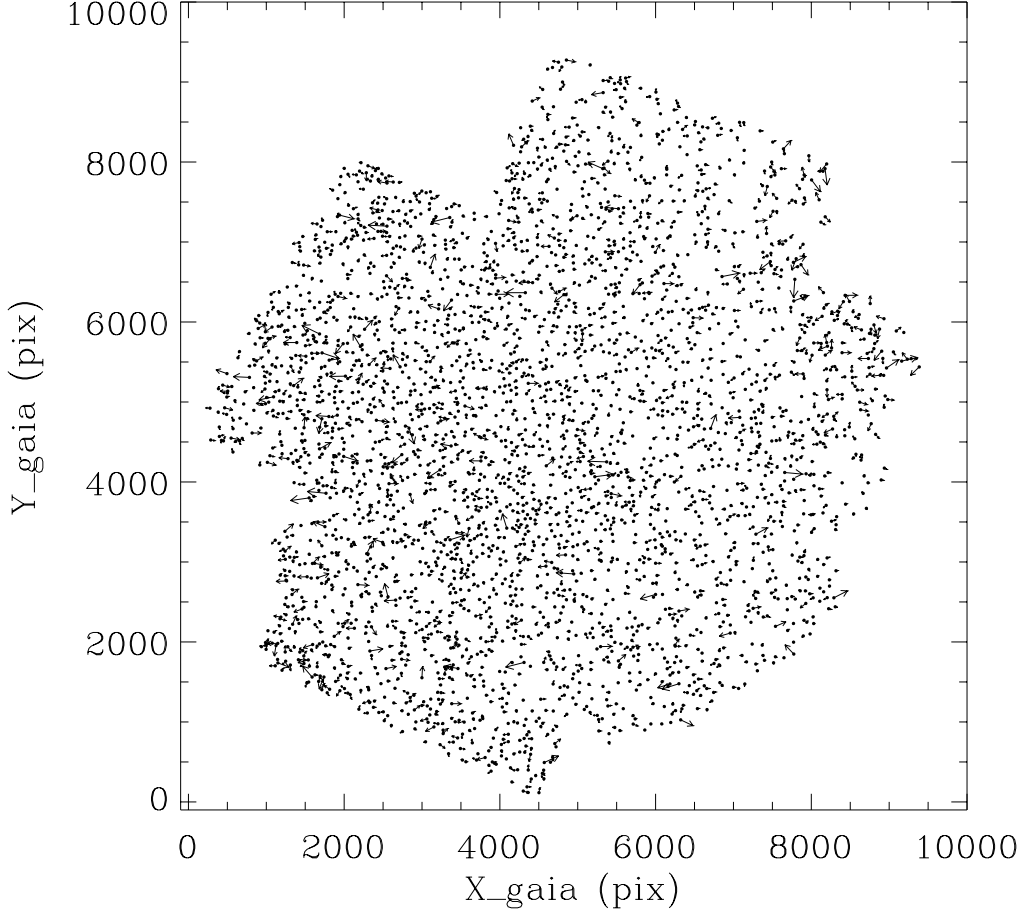


Fig. 8.— Differences between the Gaia and HST coordinate systems after accounting for global shift, rotation and scale. Residuals are clearly now random, with the largest accidental vector of ~ 0.3 pix (15 mas). Each vector is magnified by factor of 1000. The tangent-plane positions X and Y are in the units of the ACS/WFC pixels.

5. Conclusion

The first Data Release of the Gaia mission, based only on the first year of observations, has already provided high precision and accurate positions in the vicinity of the stellar cluster 47 Tuc, which had been observed with the HST ACS/WFC and used as the standard catalog for the astrometric calibration of the HST ACS/WFC. The pixel positions of the HST catalog were compared to the Gaia DR1 “pixel” positions in order to validate the HST catalog. This comparison shows a significant distortion of the HST standard catalog. First, we found large offsets in RA and Dec , of about 129 and -717 mas , respectively. These offsets are likely due to errors in the absolute astrometry of the Two Micron All Sky Survey (2MASS, Skrutskie et al., 2006), which was the best representative of the International Celestial Reference System (ICRS) prior to the Gaia era. The global rotation between the HST and Gaia coordinate systems at the adopted reference point is about $-35''$, which is three times larger than the orientation uncertainty of the ACS/WFC. This comparison has also revealed difference between the scales of the HST standard frame and that based on Gaia, resulting in the former pixel scale being larger by fraction of 0.004 mas . This difference introduces significant offsets in positions resulting of 17 mas (0.35pix) at the far edges (4000 pixels) of frame.

In summary, we conclude that the measured offsets, rotation and scale between these two coordinate systems are significant. In order to achieve truly accurate ACS/WFC, WFC3/UVIS and IR astrometric calibrations it is necessary to apply Gaia accurate absolute astrometry. After the Gaia second Data Release with improved accuracy in positions and added proper motion, the comparison of the HST standard catalog positions in the vicinity of 47 Tuc and ω Cen should be repeated again and obtained corrections applied for the final improvement of the HST calibration fields.

Finalizing the components of low-order distortions in the HST standard catalogs and improving its accuracy down to the level of 0.5 mas or better, will simultaneously improve the HST astrometric calibration to the true level of 1 mas . Consequently, the improved astrometric calibration of the HST ACS/WFC and WFC3/UVIS instruments will finally eliminate existing uncertainties in several steps of HST images manipulation: 1) accurate stacking of various HST images taken with different dither patterns and at different orientations and inter-instruments; 2) cosmic-ray rejection with higher accuracy and precision in the drizzled-combined HST images; 3) enhancement of the spatial resolution; 4) deepening of the detection limit.

6. Recommendations

The detailed analysis of the comparison between Gaia DR1 and the HST standard field in the vicinity of 47 Tuc is similar to the examination of the HST standard catalog in vicinity of ω Cen. The lack of data in the latter standard field due to the very high density of stars forces us to postpone the comparison until future Gaia data releases became available. The methods and codes presented here will be used to investigate in more detail the HST standard catalogs after the release of Gaia DR2. The detailed analysis of the geometric distortion of the two major HST imaging instruments *w.r.t.* Gaia DR2 will improve the HST astrometric calibrations down to 0.5 mas or better.

7. Acknowledgments

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