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An Improved Distortion Solution for WFPC2

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Abstract. This is a brief account of work that is published in detail elsewhere. We have derived a greatly improved set of distortion corrections for the individual chips of WFPC2. We also track the relative positions of the chips with time. We end with a description of interactions between distortion and scale that we do not understand.

1. Introduction

Most of this discussion will describe our recent redetermination of the geometric distortion corrections needed for WFPC2 images. We begin, however, with the motivation for this study.

Astrometry has two parts. One is the measurement of good positions that are free of systematic measuring errors; the other is the combination of positions measured in different images. The first, the measurement of positions, we discussed two years ago (Anderson & King 2000). The essence of the methods described there is to use as many stars as possible to derive an extremely accurate PSF. We iterate between improving the individual positions from which the PSF is created, so as to fit them together correctly, and improving the PSF, so as to get a better set of positions next time round. The demon to be exorcised is pixel-phase error, i.e., a systematic position error that depends on how each star is centered with respect to pixel boundaries. That is the basic purpose that our accurate PSF-building accomplishes.

The other part, the combination of positions measured on different images, usually in different dither positions and sometimes in different orientations, is much more complicated. It always requires a transformation from the coordinate system of one image to that of another image, and here is where distortion gets in the way.

The problem is that in order to derive the transformation from one image frame to another, one has to use the positions of a number of stars in each image, to derive a linear transformation between them. But if the distortion has not been totally removed, the true relationship will not be linear, because when the same star falls in different places in two images, these positions suffer different distortions. The non-linearities of course grow with separation in the image, so that what we are forced to do is to derive a separate transformation for each individual star, from the positions of other stars in its immediate neighborhood. But the larger the distortions that remain, the smaller is the set of neighbors that we can use, and the accuracy of the transformation suffers. Ideally, we would like to remove all the distortion, so as to be able to use a single global transformation over

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the whole image. Unfortunately that still is not possible, but minimizing the remaining distortion allows us to use more surrounding reference stars and therefore make better transformations.

That is our own interest in improving distortion corrections. But we should also note that it is only in the rich globular-cluster fields of our projects that one can use the local-transformation work-around; in sparse fields, astrometry is completely at the mercy of the distortion correction. Thus the work that we describe here not only serves our own needs but also contributes to the public welfare.

The summary that we present here will be quite brief, however, because by the time this account appears, our work will have been published in detail (Anderson & King 2003, AK03).

2. The Data Set

For the basic distortion solution we had an excellent data set available. In the so-called inner calibration field of ω Centauri there are 80 exposures with F555W, at all sorts of orientations. The variety of orientations turns out to be crucial, because when overlapping images are all at the same orientation there is no way of solving for the part of the distortion that consists of skewing.

3. The True Nature of Distortion

One tends to think of distortion as a problem that consists only of non-linearities, but that is not so. As we will see, a considerable part of the improvement that we make in the distortion correction for WFPC2 is the discovery of a hitherto unrecognized skewing in the PC chip.

This recognition of skewing as a distortion that is mathematically linear leads in turn to a consideration of what kinds of linear transformations should be used in various situations. The distinction that we make is that on the one hand we combine positions by using full 6-parameter linear transformations, in order to get the coordinate systems of the two images to match as well as possible, under conditions where some distortion may remain.

If, on the other hand, we have two coordinate systems each of whose star positions are completely distortion-free, then these systems are related by a 4-parameter transformation. The parameters represent a translation, a rotation, and a scale change. In AK03 we refer to this as a conformal transformation, because within the whole class of linear transformations, it is the sub-class that can be characterized as angle-preserving. In that paper, in fact, we make this an operational definition of undistorted images—that positions of stars in any pair of undistorted images can be related by a conformal transformation—and we apply this definition quite literally in our search for a set of corrections that will render every image undistorted.

4. The Method of Solution

For the actual solution, the conventional approach would be to take a set of overlapping images and do a least-squares solution for the positions of all the images relative to each other, and at the same time for the distortion coefficients that get the images best to conform with each other. But we disliked the black-box aspect of this, so we chose instead to start by applying the best distortion corrections that we had, and then examining all the position residuals of individual star images from the mean position of that star, as a function of location in the chip, in order to see empirically what further correction was needed. For all the thousands of stars, in all the overlaps between 80 sets of 4 chips each, we had more than a million residuals, and there were enough of them in each small region of the chip that we could see directly what the mean distortion was there. After a few iterations, we had the best distortion correction that we could get.

Another aspect of our distortion solution is important to point out: there is a separate solution for each chip. Although all four have to be solved for simultaneously, because so many of the overlaps are from one chip to another, the distortion solution for each chip is independent of the others. To put this in another way, we did *not* look for a meta-chip solution, in which all four chips are forced into the same coordinate system. The chips have moved with respect to each other, over the years, so that there is no way in which we could have accommodated our multi-year data set in a single fixed system. Thus we treated the chips separately, and looked at their relative positions later, as a separate problem.

5. The Results

Like previous corrections, ours is a third-order polynomial. We believe that it is now good to 0.01–0.02 pixel within each chip. We should note that our solution excluded, and should not be applied to, pixels in the first 100 rows and columns of each chip. We found that these are badly behaved, presumably because the OTA spherical aberration spills over the edges of the reflecting pyramid. At first glance the errors look smaller for the WFs than for the PC, but in both cases they actually run around 1 mas.

The big surprise was that there is a quite appreciable skewing in the PC, amounting to about half a pixel over the length of its edge. Previous solutions, which did not have overlaps that were rotated, had been unable to detect this; and the fact that the otherwise excellent solution by Casertano & Wiggs (2001) was a meta-chip solution caused the skew in the PC somewhat to infect the corrections in the other chips.

For the relative positions of the chips we used the outer calibration field in ω Centauri, which has more than 1200 observations spread from 1994 to 2002. We used successive overlaps in a rich set of observations at a single epoch to find the positions of all the stars in a meta-chip coordinate system, and then simply matched each chip in each exposure to this system, to find where it was in relation to a fiducial point in WF3. The result is a graph of positions against time for each of the other chips. These are good only to about a tenth of a pixel, so relative positions within a chip can be measured with much better accuracy than positions from one chip to another.

All of this is described in detail in our paper (AK03). For the convenience of WFPC2 users there is a link to that paper in the WFPC2 web pages. On a more practical level, the web pages also have a copy of a Fortran subroutine which, when given the name of a chip and column and row values (x, y), returns the distortion-corrected position.

6. Desiderata

Unfortunately our distortion solution is not as good as we would wish. An accuracy of 0.01 pixel is better than ever before, but in many of our measurements we need accuracies of 0.001 pixel or better, so we are still unable to make transformations that extend over more than a small fraction of a chip. The problem is that we have been unable to find distortion changes that are independent of, or even that allow for, the scale changes that are constantly taking place from one *HST* image to the next.

The scale changes have three contributors. Two of them come from changes in the OTA focal length. Occasional adjustments are made in the position of the secondary, but our main problem comes from orbital breathing, which produces a range of scale change of about 10^{-4} around each orbit, in the WFs, and about half that much in the PC. (The difference is a simple consequence of the Gaussian optics of the transfer systems.)

The third contributor to scale change is velocity aberration. An appendix to our paper (AK03) shows that velocity aberration produces a scale change that is v/c times the cosine of the angle of the target from the plane of motion. Since v/c is 10^{-4} for the motion of the Earth around the Sun, and the *HST* orbital velocity is also about 1/4 as large as that, it is clear that velocity aberration can also be an important contributor to scale changes.

For the study of scale changes due to breathing we had available another valuable data set, what we like to refer to as the "Gilliland stare," 8 days of nearly continuous CVZ imaging of 47 Tucanae (GO-8267).

There are interesting correlations of scale with orbital phase and with time of day. We simply do not understand them, and have not studied them further.

By far the most intriguing correlation we found was between scale and distortion. What we did was to look at the positional residuals in each image, and fit them with Legendre polynomials (just for convenience, because they are quicker to fit than powers). We find that each of the coefficients correlates closely with scale, so that there is a clean, systematic change in distortion as the scale changes. (By the way, this is not just a question of whether we apply the distortion correction before or after the scale change; we tried it both ways, and it makes hardly any difference.) The frustrating thing about this scale-dependent distortion is that whereas we could in principle correct for this within the Gilliland data set where we know all the relative scales, for a randomly chosen image of another field we do not know where we are in the range of scales, so that we don't know how to correct for this last bit of distortion. This is only a small effect—about a hundredth of a pixel from center to edge—but we wish we could fix it. Again, a hundredth of a pixel does not sound like much, but in some of our current work it is just not good enough accuracy.

We do not propose to study these scale effects any further, but hope that some one else will. We will happily make available the scale factors of the more than 1200 Gilliland images.

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