Chapter 3

The statistics of disk and bulge parameters

Abstract. The statistics of the fundamental bulge and disk parameters of galaxies and their relation to the Hubble sequence were investigated by an analysis of optical and near-infrared observations of 86 face-on spiral galaxies. The availability of near-infrared $K$ passband data made it possible for the first time to trace fundamental parameters related to the luminous mass while hardly being hampered by the effects of dust and stellar populations. The observed number frequency of galaxies was corrected for selection effects to calculate volume number densities of galaxies with respect to their fundamental parameters. The main conclusions of this investigation are:

1) Freeman’s law has to be redefined. There is no single preferred value for the central surface brightnesses of disks in galaxies. There is only an upper limit to the central surface brightnesses of disks, while for lower central surface brightnesses the number of galaxies per volume element decreases only slowly as function of the central surface brightness.

2) The Hubble sequence type index correlates strongly with the effective surface brightness of the bulge, much better than with the bulge-to-disk ratio.

3) The disk and bulge scalelengths are correlated.

4) These scalelengths are not correlated with Hubble type. Hubble type is a lengthscale-free parameter and each type therefore comes in a range of magnitudes (and presumably a range of total masses).

5) Low surface brightness spiral galaxies are not a separate class of galaxies. In a number of aspects they are a continuation of a trend defined by the high surface brightness galaxies. Low surface brightness galaxies are in general of late Hubble type.

1 Introduction

The light of a spiral galaxy is dominated by two components, the disk and the bulge. The basic difference between these components lies in their support against gravitational collapse. The disk is almost completely rotationally supported, while the bulge is for some fraction also pressure supported. At least two parameters are needed to describe the light distribution of each of these components: a surface brightness term and a spatial scaling factor. The fundamental parameters of the disk are usually expressed in central surface brightness ($\mu_0$) and scalelength ($h$), while the bulge parameters are expressed in effective surface brightness ($\mu_e$) and effective radius ($r_e$). These fundamental parameters were determined for a large statistically complete sample of galaxies by de Jong (1995a, Paper II). The distributions of the fundamental parameters are still poorly known and their statistics are investigated in this paper with some emphasis on three relationships: 1) “Freeman’s law”, the empirical relation found by Freeman (1970) indicating the constancy of disk central surface brightness among galaxies, 2) the number density of galaxies as a function of their fundamental parameters and 3) the relation between the fundamental parameters and Hubble classification.

1.1 Freeman’s law

One of the most remarkable results presented in the classical paper of Freeman (1970) was the apparent constancy of the $B$ passband $\mu_0$ of spiral galaxies. For a subsample of 28 (out of 36) galaxies he found $\langle \mu_0 \rangle = 21.65 \pm 0.3 \ B$-mag arcsec$^{-2}$. If the central $M/L$ is approximately constant among galaxies, this translates directly into a constant central surface density of matter associated with the luminous material.

Several authors have tried to explain this result. It has been argued that ignoring the contribution of the bulge to the light profile could produce the effect (Kormendy 1977; Phillipps & Disney 1983; Davies 1990). Freeman (1970) did not decompose the luminosity profiles in a bulge and disk, but fitted a line to the linear part of the luminosity profile plotted on a magnitude scale. This linear part of the profile could be contaminated by bulge light. With their models Kormendy (1977) and Davies (1990) show that the central surface brightness of low surface brightness disks will be overestimated by this procedure because of the extra bulge light near the center. The central surface brightnesses of high surface brightness disks with a short scalelength are underestimated; because of the small disk scalelength the bulge light dominates the luminosity profile again in the outer region, but with a longer scalelength and a lower surface brightness than the disk. Several arguments can be raised against this interpretation (see also Freeman 1978): 1) even with bulge light included the result is still important, 2) many later type galaxies hardly have a bulge, but the effect is still present (van der Kruit 1987), 3) in samples where proper decomposition techniques are used the effect is still found, although with a larger dispersion (Boroson 1981), 4) a limited range in bulge parameter space was explored in the models mentioned above, which might not be representative of the bulges in spiral galaxies.
Dust extinction has also been proposed as an explanation for the constancy of \( \mu_0 \) (Jura 1980; Valentijn 1990). If galaxies are optically thick in the \( B \) passband, one is only looking one optical depth into the galaxies and always observes the same outer layer. This removes the inclination dependence from the Freeman relation, but leaves the unsolved problem of why all galaxies should have the same surface brightness at optical depth equal to one.

Freeman established his relation in the \( B \) passband where the light of galaxies is dominated by a very young population of stars, which make up only a few percent of the stellar mass. Of all commonly used passbands the light of the massive old stellar population is relatively the most important in the near-infrared (near-IR) \( K \) passband used here. The \( K \) passband has the additional advantage that the extinction by dust is strongly reduced. The \( K \) passband is therefore best suited to trace the fundamental parameters of the luminous mass. However, other passbands have been used as well in this study to investigate the wavelength dependence of the bulge and disk parameters due to dust and population effects.

De Vaucouleurs (1974) was one of the first to suggest that the constancy of \( \mu_0 \) might result from a selection effect. This was later quantified by Disney (1976) and Allen & Shu (1979). Catalogs of galaxies have usually been selected by eye from photographic plates using some kind of diameter limit. One might therefore select against very compact galaxies with a high central surface brightness, because these have small isophotal diameters. Likewise, galaxies with a very low surface brightness might have been missed due to the lack of contrast with the sky background. Disney & Phillips (1983; see also Davies 1990) define a visibility for a galaxy, which enables one to correct a sample for these selection effects if one has made a careful initial sample selection.

### 1.2 Bivariate distributions

Correcting for selection effects is in fact trying to determine from the observed statistics how many galaxies there are per unit volume with a certain property. More than one property can be used in determining such a distribution per volume. One needs at least two parameters to characterize the exponential light profile of a disk dominated galaxy and a bivariate distribution function of both disk parameters is a more general statistical description of galaxy properties than a one parameter function. The diameter, the central surface brightness and the luminosity distribution functions of galaxies are integrations of this bivariate distribution in a certain direction. In this process information is lost and the bivariate distribution function is therefore more useful in studies of deep galaxy counts and provides more constraints on theories of galaxy formation and evolution than its one dimensional counterparts.

Bivariate distribution functions of galaxies have been determined only a few times before (Choloniewski 1985; Phillips & Disney 1986; van der Kruit 1987, 1989; Saunders et al. 1990; Sodrè & Lahav 1993). Even though different fundamental parameters are used, almost all (except Saunders et al.) of these distributions describe fundamentally the same thing in different ways. These studies were performed in the \( B \) or comparable passbands, which is, as mentioned before, not the wavelength most suited to study global fundamental properties of galaxies.

### 1.3 Morphological classification

For classification of spiral galaxies on the Hubble sequence three principal discriminators are used: 1) the pitch-angle of the spiral arms, 2) the degree of resolution of the arms (into \( H \) regions, dust lanes and resolved stars) and 3) the bulge-to-disk (B/D) ratio. In his detailed description of the Hubble sequence, Sandage (1961) indicates that the B/D ratio is the weakest discriminator unless galaxies are seen edge-on. He finds clear mismatches in type between classifications using items 1) and 2) and classifications using item 3). Another factor hampers the use of B/D ratio for classification of early spirals. On the photographs used for classification the central region of an early spiral galaxy is normally overexposed in order to show clearly the faint spiral structure.

Still, the B/D ratio is often assumed to be the principle underlying the Hubble sequence, even though a tight correlation between classification and measured B/D ratios was never found. The measurements indicate at best a trend (e.g. Simien & de Vaucouleurs 1986; Andredakis & Sanders 1994) and the discrepancies between B/D ratio and Hubble type have been attributed to two sources of error. First there is the uncertainty in classification. Comparisons of Hubble types given by different classifiers show an rms uncertainty in type index of order 2 T-units (Lahav et al. 1995). The second source of error is the uncertainty in the bulge/disk decomposition, due to, among other things, the mathematical peculiarities of the widely used \( r^{1/4} \) bulge law (de Vaucouleurs 1948).

### 1.4 Outline

The main goal of this investigation is to determine the nature of the Freeman law. In order to address the problems concerning the Freeman law, a large sample of face-on spiral galaxies was carefully selected and surface photometry was obtained in the \( K \) passband as well as in several other passbands. A large number of other global and structural parameters of the galaxies were determined in this investigation and their nature is also explored in this paper.

The remainder of this article is organized as follows. The data set and the extraction of the observed bulge and disk parameters are briefly described in Section 2. The corrections to the observations in order to calculate number distributions are described in Section 3 and these distributions are presented for the \( B \) and the \( K \) passband in Section 4. The relations found are discussed within the context of the three main points of interest (Freeman’s law, bivariate distributions and Hubble sequence) in Section 5. The conclusions are summarized in Section 6.

#### 2 The data

In order to examine the parameters describing the global structure of spiral galaxies, 86 systems were observed in the
3 Corrections

The observed bulge and disk parameters determined in Paper II have to be corrected for all kinds of systematic effects. These corrections are often uncertain but necessary. One can only expect that they are at least in a statistical sense correct.

## 3.1 Galactic foreground extinction

The measurements of brightness and surface brightness were corrected (unless stated otherwise) for Galactic foreground extinction according to the precepts of Burstein & Heiles (1984) and the actual B passband extinction values were adopted from the RC3. The Galactic extinction curve of Rieke & Lebofsky (1985) was used to convert these B passband extinction values to other passbands. The sample galaxies were selected to have a Galactic latitude larger than 25°; the extinction correction is in general small and gets smaller for the longer wavelength passbands. The average correction is 0.14 B-mag and the largest correction is 0.68 B-mag, which translates into 0.06 K-mag.

## 3.2 Inclination corrections

Since Valentijn (1990) reopened the debate of optically thin versus optically thick spiral galaxies, inclination corrections for surface brightness have become less trivial. A simple equation for correcting surface brightnesses for inclination effects, taking internal extinction into account, has the form

\[ \mu^I = \mu - 2.5C \log(a/b), \]  

where \( a/b \) is the major over minor axis ratio of the galaxy and \( C \) the internal extinction parameter, which takes values \( 0 \leq C \leq 1 \). Fully transparent galaxies are described by \( C = 1 \), while the case \( C = 0 \) describes the optically thick ones.

It is unlikely that the inclination correction indeed takes such a form in the optical passbands, as extinction in the optical passbands is for a considerable fraction caused by scattering and not just by absorption alone. Light will be scattered preferably from edge-on directions to face-on directions, which means that extinctions will seem to be higher for edge-on than for face-on galaxies. On top of that, certain configurations of dust and stars can behave optically thin in an inclination test, while they may in fact be completely opaque. A clear example of this is a very thin layer of optically thick dust between a thicker slab of stars. It is not trivial to produce a better description as there are too many unknowns and \( C \) itself may be a function of galactic radius (see e.g. Giovanelli et al. 1994; Byun et al. 1994). Therefore Eq. (1) is used as a working hypothesis. However, for a face-on selected sample such corrections are small. The average correction for the sample examined here is 0.26 mag arcsec\(^{-2}\) when \( C = 1 \), with a maximum of 0.60 mag arcsec\(^{-2}\) if \( b/a = 0.58 \).

## 3.3 Distances

The distances to the observed galaxies were calculated using a Hubble flow with an \( H_0 \) of 100 km s\(^{-1}\) Mpc\(^{-1}\), corrected for infall into the Virgo cluster using the 220 model of Kraaikorteweg (1986). This model assumes that the Local Group has an infall velocity of 220 km/s towards the Virgo cluster and describes the motions of the galaxies around the cluster by a non-linear flow model. The \( V_{GSR} \) velocities needed for this model were calculated from the \( V_{GSR} \) velocities listed in the
RC3, which are also tabulated in Paper I. The nearest galaxy is at 6.2 Mpc, the most distant galaxy is at 82.5 Mpc. The peculiar velocities of galaxies were assumed to be on average 200 km/s in the line of sight, which introduces an error ($\sigma_d$) of 2 Mpc in the distance estimates. The distribution of distances is displayed in Fig. 1, which shows a small excess of galaxies at ~45 Mpc because of an extension of the Pisces-Perseus supercluster. The relationships presented in this study are very little affected when other infall and flow models are used to calculate distances.

### 3.4 Selection correction

The physically relevant quantities are not the observed numbers of galaxies with a certain property, but the frequency of galaxies with a certain property in a volume. Therefore, the fact that a galaxy is included in the sample has to be linked to the statistical probability of finding such a galaxy in a certain volume. The galaxies in the sample were selected to have UGC red major axis diameter ($D_{maj}^{maj}$) of at least 2 arcmin. This creates a selection bias against galaxies with low surface brightness and/or small scalelengths, as they appear smaller on photographic plates. The distances ($d$) to the observed galaxies and their angular diameters ($D_{maj}$) are known and the maximum distance at which galaxy can be placed, while still obeying the selection criteria, can be calculated ($d_{max} = dD_{maj}/D_{maj}^{maj}$). A galaxy can only enter the sample if it lies in a spherical volume which has this maximum observable distance as radius. Turning this argument around, one can expect on statistical grounds that a selected galaxy samples a spherical volume with a radius equal to its maximum observable distance (a more formal discussion can be found in Falten 1976). The volume sampled by a galaxy in a diameter limited sample is thus

$$V_{max} = \frac{4\pi}{3}(d_{max})^3 = \frac{4\pi}{3}(dD_{maj}/D_{maj}^{maj})^3.$$  (2)

Following the previous line of reasoning, an estimate for the average number of galaxies in a unit volume obeying a certain specification ($S$) for a complete sample of $N$ galaxies is

$$\Phi(S) = \frac{N}{N} \sum_{i=1}^{N} S_i^i / V_{max}^i,$$  (3)

where $i$ is summed over all $N$ galaxies in the sample and $S_i^i = 1$ if the specification is true for galaxy $i$ and $S_i^i = 0$ if false. The error in $\Phi(S)$, assuming Poisson statistics in a homogeneous universe and considering the uncertainties in the distances, can be calculated by

$$\sigma_{\Phi(S)}^2 = \frac{1}{N} \sum_{i=1}^{N} (S_i^i / V_{max})^2 + \frac{1}{N} \sum_{i=1}^{N} (3S_i^i / dV_{max})^2.$$  (4)

There is always a chance that a member of a peculiar class of galaxy happens to be nearby and gets a lot weight in Eq. (3) and this volume correction can therefore only be applied to large samples. One must ensure that a large enough volume of space is sampled so that galaxies are randomly distributed in space. Figure 1 shows that the sample mainly traces the local density enhancement, as large scale structures in the universe have scales of order 50 Mpc. Equation 3 should therefore be used with care, because the number of galaxies with small intrinsic diameters will be overestimated relative to the larger ones due to the local density enhancement. The average number of galaxies per Mpc$^3$ calculated with Eq. (3) might be more representative of the local environment than of the mean cosmological values. Still it is a useful equation to observe general trends in bivariate distributions and to compare results obtained from different passbands.

Other methods to correct distributions for selection effects have been advocated, because they take spatial density fluctuations into account (for an overview see Efstathiou et al. 1988). These methods assume that the intrinsic distribution function is independent of position ($x$) in space, so that we can write $\Phi(S) = \phi(S)\rho(x)$, thereby losing the absolute calibration of the number density. These methods all assume a clear relation between the distribution parameter(s) and the limiting selection parameter(s). This is not the case for the current investigation. A diameter limit is not trivially linked to the central surface brightness distribution, certainly not when a different passband is used for the selection and the distribution.

The correction of Eq. (3) is only valid if a particular galaxy would have been measured at the same intrinsic (as opposed to angular) diameter, had it been at a different distance. In Paper I it was shown that this is probably the case for the UGC galaxies with type index $T \leq 6$. For later types the situation is less clear, there is a too short a range in diameters to check and it must be assumed that for late-type systems the same type of galaxy is measured at the same intrinsic diameter at different distances. Under this assumption it is not important that the UGC eye estimated diameters of late-type galaxies correspond to lower average surface brightness than that of early types (see Paper I, Fig. 11). This effect just means that there are more late-type galaxies in the sample than expected based on their isophotal

![Fig. 1. The distance distribution of the sample galaxies. For the dashed line the $V_{GSR}$ velocities from the RC3 were used, the full line indicates the distance distribution when the velocities are corrected for Virgocentric infall.](image-url)
diameter, but their average distance will be larger so that the number of galaxies per sampled volume stays the same.

The volume correction of Eq. (3) can be used to calculate number density distributions for all passbands, as long as the red UGC diameters are used to calculate the \( V_{\text{max}} \). The distribution of any galaxy parameter \( S^i \) can be determined in any passband; the use of the red UGC diameters in Eq. (3) ensures the correction for the intrinsic selection effects of the whole sample.

Next to the diameter limit, there are two more selection criteria defining the sample. The selection was limited to 12.5% of the sky and only galaxies with \( h/a > 0.625 \) were used, which is only 37.5% of all possible random orientations. Equation 3 was corrected for these selection criteria. A correction was also applied for the fraction of galaxies for which no (photometric) data was available in a certain passband. All these corrections were made under the assumption that the incompleteness had no correlation with the investigated parameters.

Equation (3) can only be applied when the sample is complete. The statistical completeness of the sample can be tested with the \( V/V_{\text{max}} \)-test (Paper I). The \( V/V_{\text{max}} \) of a galaxy is the spherical volume associated with the distance of a galaxy divided by \( V_{\text{max}} \) as defined in Eq. (3), thus for a galaxy in this diameter limited sample \( V/V_{\text{max}} = \left( D_{\text{maj}}/D_{\text{maj}}^\text{lim} \right)^3 \). For objects distributed randomly in space the average value of \( V/V_{\text{max}} \) should be \( 0.5 \pm 1/\sqrt{2N} \), where \( N \) is the number of objects in the test. For the current sample \( V/V_{\text{max}} = 0.57 \pm 0.03 \) and therefore there are slightly too many galaxies with a small angular diameter in the sample. The original sample of 368 galaxies from which the current subsample was selected had a \( \langle V/V_{\text{max}} \rangle = 0.496 \pm 0.015 \) (Paper I). Subsequent selection depended only on the position on the sky and therefore the excess of small diameter galaxies is probably caused by the density enhancement of the Pisces-Perseus supercluster, which gives some extra galaxies at the diameter selection limit. This might give some extra high surface brightness and/or large scalelength galaxies in the sample above the cosmological mean, because galaxies have to be intrinsically large to be included in the sample being at the distance of the Pisces-Perseus supercluster.

In a recent paper Davies et al. (1994) argued that the sample used by van der Kruit (1987) was incomplete in a magnitude \( V/V_{\text{max}} \)-test. They argued that a hidden magnitude limit had influenced the selection, so that an extra selection correction should be applied. I will follow up on this argument as the sample used here has been selected using similar criteria as van der Kruit used for his sample. ¹

There is nothing hidden about a magnitude selection effect for a diameter limited sample. On the contrary, it is expected. For galaxies with a certain absolute magnitude \( M \)

¹ Davies et al. (1994) also indicate that van der Kruit’s sample becomes incomplete for low surface brightnesses at \( \mu_0 > 22.3 \) as \( \langle V/V_{\text{max}} \rangle = 0.35 \pm 0.08 \). I would like to note that this might just be a statistical fluctuation of low number statistics, as \( \langle V/V_{\text{max}} \rangle = 0.41 \pm 0.10 \) for \( \mu_0 > 22.5 \) and \( \langle V/V_{\text{max}} \rangle = 0.46 \pm 0.13 \) for \( \mu_0 > 22.7 \), and thus for even lower surface brightnesses the sample is in the statistically complete range of \( \langle V/V_{\text{max}} \rangle = 0.5 \).

**4 The distribution of disk, bulge and bar parameters**

In this section I investigate the distributions of the structural parameters of the different galaxy components as a function of morphological type and of each other. First the structural parameters of the disk and bulge are examined independently. In the final subsection, the relationships between disk and bulge parameters are investigated. The distributions of bulge and disk parameters are corrected for selection effects to yield volume number densities.

**4.1 The disk parameters**

Figure 2 indicates some aspects of the completeness and selection effects of the sample. It shows the distribution of the observed central surface brightnesses versus scalelength as
obtained from the 2D fits of Paper II. The $R$ passband values are plotted, because these values are most closely related to the (red UGC diameter) selection criteria. The dotted line indicates the selection limit for a diameter cutoff at 2 arcmin at a surface brightness of 24.7 $R$-mag arcsec$^{-2}$ for a perfect exponential disk. The 24.7 $R$-mag arcsec$^{-2}$ is the average surface brightness at which the UGC red diameters were determined (see Paper I). As mentioned in Paper I, not all UGC galaxies had their diameters estimated at the same isophote level. This explains why there are some galaxies to the left of the selection line in Fig. 2. If all galaxies had the same scalelength, the number of galaxies expected in the sample will decrease as $h_{ap}^3$ and therefore it is not surprising that there are hardly any galaxies in the sample below 22 $R$-mag arcsec$^{-2}$. Obviously no galaxies can enter the sample with $\mu_0$ fainter than $\sim$24.7 $R$-mag arcsec$^{-2}$.

Let us now look at the central surface brightness as function of morphological type (Fig. 3). Apparently the galaxies from type T = 1 to 6 have on average the same $\mu_0$, but with a large scatter. The later types have on average a significantly lower central surface brightnesses and they might be classified as late types just because they are low surface brightness (LSB) systems. It can be readily seen that this difference between early and late-type galaxies increases when going from the $B$ to the $K$ passband. This indicates that disks of the later type spirals are bluer than the disks of the early ones, but the discussion on the colors of these galaxies is postponed to Paper IV in this series (de Jong 1995b). The average $\mu_0$ values were calculated for three morphological type bins indicated by the horizontal bars in Fig. 3 as well as for the total sample. The values with their standard deviations are tabulated in Table 1.

The average $\mu_0$ values were also calculated with an inclination correction according to Eq. (1) with values for $C = 0.5$ and $C = 1$ (semi transparent and completely transparent behavior). The results can also be found in Table 1. The standard deviations on the average $\mu_0$ values are slightly smaller for $C = 1$, and even though it is a small effect, it is persistent for all subgroups and all passbands. The main result is of course a shift in the mean central surface brightness of the disks. For all remaining plots an inclination correction with $C = 1$ will be used.

The distribution of the other disk parameter, the scalelength ($h$), as function of type is shown in Fig. 4. There is no trend of $h$ with type and there is a large range in scalelengths. There might be a lack of late-type galaxies with small scalelengths, but this can probably be attributed to a selection effect: the selection criteria are heavily biased against LSB galaxies with small scalelengths. The scalelengths are smaller in the $B$ passband than in the $K$ passband (discussion in Paper IV).

The information of Figs 3 and 4 are combined in Fig. 5. This figure shows that there is an upper limit in the ($\mu_0, h$)-plane, as there are no galaxies with large scalelengths and high central surface brightnesses. This cannot be caused by selection effects, large bright galaxies just cannot be missed in a diameter selected sample. This upper limit has been noted before by Grosbøl (1985). The upper limit partly follows the line of constant total disk luminosity, as indicated by the dashed line in Fig. 5. Note that the Tully-Fisher relation (1977, hereafter TF-relation) implies that this is also a line of constant maximum rotation speed of the disk. There is also an upper limit to the central surface brightness at about 20 $B$-mag arcsec$^{-2}$ (16 $K$-mag arcsec$^{-2}$). Again galaxies brighter than these limits are hard to miss because of selection effects.

Late-type galaxies have lower central surface brightnesses in Fig. 5, but the early and intermediate types show no segregation. The scalelengths also gives no segregation according to type. Very few late-type galaxies with very short scalelengths were selected, but as shown before, late-type galaxies have lower surface brightnesses and the selection biases against galaxies with low surface brightness and short scalelengths are large. These biases are indicated by the dotted lines in Fig. 5. To the right of these lines the sample should be complete to the indicated distance. The lines are calculated under the assumption that all galaxies have perfect exponential disks with the same color at the selection radius ($B-R = 1.3$, $R-K = 2.5$) and that the selection limit is at 2' diameter at the 24.7 $R$-mag arcsec$^{-2}$ isophote (as in Fig. 2). Although these assumptions are not valid for an individual galaxy, the dotted lines help to estimate the selection effects; the galaxies near the 50 Mpc line had about 125 times more chance of being included in the sample than the galaxies near the 10 Mpc line! The fact that the number density of objects does not decrease by 125 from one line to the other already indicates that there are many more “small” galaxies per volume element than “large” galaxies.

The distribution of the absolute magnitude of the disk ($M_{disk}$) against type (Fig.6) can also be deduced from Figs 3 and 4 ($M_{disk} \propto \mu_0 = 2.5 \log(2\pi h^2)$, no inclination dependent extinction correction was applied). As scalelengths show little correlation with type, the distribution of disk magnitudes reflects the distribution of the central surface brightness against type. There was no apparent segregation according to bar classification in Figs 3, 4, 5 and 6.

So far, only the observed distributions were presented, but the distributions per volume are of more importance. Therefore the volume correction as described in Section 3 was applied. The correction transforms Fig. 5 into the bivariate distribution in the ($\mu_0, h$)-plane presented in Fig. 7. This is a representation of the true number distribution of spiral galaxies per volume element of one Mpc$^3$ with respect to both disk parameters. The magnitude and $\mu_0$ upper limits noticed in Fig. 5 are also present here. We are dealing with low number statistics now, which is reflected in the erratic behavior of the distribution. The uncertainty increases in the direction of small scalelength and low surface brightness. These galaxies have so small isophotal diameters that they really have to be nearby to be included in the sample and such a small volume is sampled that statistics are working against us. For example if the true volume densities in the (17 $K$-mag arcsec$^{-2}$, 1 kpc) and (21 $K$-mag arcsec$^{-2}$, 1 kpc)-bins are equal, the chance of observing a galaxy in the last bin would be 0.5. If there had been such a galaxy in the sample, a lot of weight would have been given to it. In short, the distributions are not well sampled in the low surface
Fig. 3. The Galactic extinction corrected central surface brightness of the disks as function of morphological RC3 type. The crosses show the values averaged over the bins indicated by the horizontal bars. The vertical bars indicate the standard deviations of the mean values.

Fig. 4. The scalelength of the disk as function of morphological type.

Fig. 5. The scalelength of the disks versus the central surface brightness. Different symbols are used to denote the indicated morphological type ranges. Exponential disks with equal absolute luminosity of indicated magnitude are found on the dashed line. Equality lines of other magnitudes lie parallel to the dashed line. The dotted lines indicate the selection limits for all exponential disk galaxies closer than 10 and 50 Mpc respectively, under the assumptions made in the text.
The number density of galaxies decreases sharply with
brightness, small scalelength region. No galaxies were selected
in this region, but the traced volume is also very small. The
dominant type of spiral galaxy has a scalelength of about 1 kpc
and a central surface brightness of 21.5 mag arcsec$^{-2}$
(16 $K$-mag arcsec$^{-2}$). The distributions are remarkably
flat for the total sample. The distributions will
be narrowed for central surface brightnesses fainter than 20
$B$-mag arcsec$^{-2}$ (19 $K$-mag arcsec$^{-2}$) and the $h$
distributions should not be trusted for scalelengths smaller than 1 kpc.
The undersampling in the $\mu_0$ distribution is considerably reduced
when only types earlier than T=6 are used. Disks of late-
type galaxies are bluer, which makes the overall distribution
narrower in $B$ than in $K$. The distributions are not limited by
selection effects at the bright end, even if one assumes there is
an upper limit to the total luminosity of a galaxy (see Fig. 5).
The number density of galaxies decreases sharply with $\mu_0$
brighter than 20 $B$-mag arcsec$^{-2}$ (~16 $K$-mag arcsec$^{-2}$). At
the faint end a limited volume is sampled, and Fig. 5 indicates
that the sample is biased against galaxies with a $\mu_0$ fainter than
23 $B$-mag arcsec$^{-2}$ even for galaxies with scalelength larger
than 1 kpc. Obviously galaxies with central surface brightness
fainter than 26 $B$-mag arcsec$^{-2}$ could never enter the sample.
The distributions of $\mu_0$ in Fig. 8 could be slightly higher at
the faint end and should probably be extended to much lower
surface brightnesses.

The volume corrected distributions of the logarithm of the
scalelengths (Fig. 9) show first a small increase of galaxies to
scalelengths of about 1 kpc. This is probably caused by the
undersampling effect at low surface brightnesses and small
scalelengths. For scalelengths larger than 1 kpc we notice a
steady decline of about a factor 100 in one dex. There is no
segregation with morphological type.

The most important results obtained in this subsection
are as follows. There is a large range in disk central surface
brightnesses among galaxies, mainly due to the lower surface
brightnesses of late-type galaxies. The range decreases slightly
when a transparent inclination correction is used. Selection
effects are very significant in determining number density
distributions and after correcting for these effects there is no

### Table 1. The average Galactic extinction corrected central surface brightnesses for different inclination corrections (Eq. (1)) and type index bins. $C = 0$ corresponds to an optically thick disk, $C = 0.5$ to a semi transparent disk and $C = 1$ to a fully transparent disk. The values are in mag arcsec$^{-2}$ with their standard deviations.

<table>
<thead>
<tr>
<th>RC3 type</th>
<th>nr.</th>
<th>$C = 0$</th>
<th>$C = 0.5$</th>
<th>$C = 1$</th>
<th>$C = 0$</th>
<th>$C = 0.5$</th>
<th>$C = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 $\leq$ T $&lt; 6$</td>
<td>61</td>
<td>21.32 $\pm$ 0.78</td>
<td>21.45 $\pm$ 0.76</td>
<td>21.58 $\pm$ 0.74</td>
<td>60</td>
<td>17.48 $\pm$ 0.71</td>
<td>17.61 $\pm$ 0.69</td>
</tr>
<tr>
<td>6 $\leq$ T $&lt; 8$</td>
<td>12</td>
<td>22.01 $\pm$ 0.75</td>
<td>22.16 $\pm$ 0.73</td>
<td>22.30 $\pm$ 0.72</td>
<td>10</td>
<td>18.34 $\pm$ 0.90</td>
<td>18.50 $\pm$ 0.90</td>
</tr>
<tr>
<td>8 $\leq$ T $\leq 10$</td>
<td>8</td>
<td>22.97 $\pm$ 0.60</td>
<td>23.12 $\pm$ 0.57</td>
<td>23.26 $\pm$ 0.55</td>
<td>7</td>
<td>20.05 $\pm$ 1.05</td>
<td>20.21 $\pm$ 1.03</td>
</tr>
<tr>
<td>all</td>
<td>81</td>
<td>21.59 $\pm$ 0.92</td>
<td>21.72 $\pm$ 0.90</td>
<td>21.86 $\pm$ 0.89</td>
<td>77</td>
<td>17.82 $\pm$ 1.08</td>
<td>17.96 $\pm$ 1.08</td>
</tr>
</tbody>
</table>
The distribution of disk, bulge and bar parameters

Fig. 7. The volume corrected bivariate distribution of galaxies in the \((\mu_0, h)\)-plane. The number density \(\Phi(\mu_0, h)\) is per bin size, which is in steps of 0.3 in \(\log(h)\) and 1 mag arcsec\(^{-2}\) in \(\mu_0\).

Fig. 8. The volume corrected distribution of the central surface brightness. The dashed line indicates the distribution for types earlier than type \(T=6\). The number density is per bin size, which is in steps of 0.75 mag arcsec\(^{-2}\) in \(\mu_0\).

Fig. 9. The volume corrected distribution of the disk scalelengths. The dashed line indicates the distribution for type earlier than \(T=6\). The number density is per bin size, which is in steps of 0.2 in \(\log(h)\).
Fig. 10. The Galactic extinction corrected effective surface brightness of the bulge as function of morphological RC3 type.

Fig. 11. The effective radius of the bulge as function of morphological type.

Fig. 12. The effective radius of the bulge versus the effective surface brightness at this radius. Different symbols are used to denote the indicated type ranges.
single preferred value for the central surface brightnesses of disks. There is an upper limit to \( \mu_0 \), but the number density distribution decreases only slowly at the faint end.

### 4.2 The bulge parameters

The same diagrams used to describe the disk parameters are now used to present the bulge parameters. The distributions of effective surface brightness are presented in Fig. 10. The effective surface brightness shows a tight correlation with type index, especially considering the uncertainty of at least 1.5 T-units (1 sigma) in type index (Lahav 1995). Almost all of the scatter can be explained by this uncertainty. This relation also holds for the \( \mu_e \) parameters obtained with the other fitting methods presented in Paper II, although with a slightly larger scatter. There is no apparent correlation of effective radius with galaxy type (Fig. 11). The relations in Figs 10 and 11 are tighter in K than in B. There are several explanations for this effect. Bulges are relatively brighter with respect to the disks in \( K \) compared to \( B \), which will make the fit routine work better. Furthermore, circumnuclear star formation and dust lanes will affect the \( B \) passband more than the \( K \) passband and make the quality of the decomposition worse. There are some galaxies in the sample with clear circumnuclear star formation and with dust lanes right down to the center. Finally, there is the effect of the Freeman Type II profiles (Freeman 1970) which is reduced in \( K \), thus making fitting easier (see Paper II). The distribution of points in the \((\mu_e, r_e)\)-plane (Fig. 12) shows no correlation. The absence of a correlation between \( r_e \) and morphological type makes the trend in the distribution of the absolute bulge magnitude \( (M_{\text{bulge}} \propto -2.5 \log(r_e^2)) \) versus type (Fig. 13) dominated by the \( \mu_e \) rather large though.

The bivariate distribution of \( \mu_e \) and \( r_e \) (Fig. 14) shows no trends. The dominant type of galaxy in our local universe has a bulge with effective radius in the range of 0.1-0.3 kpc and effective surface brightness of order 21 \( B \)-mag arcsec\(^{-2}\) \((\sim 16 \ K\)-mag arcsec\(^{-2}\)). The relation between the bulge parameters and the diameter selection criterion is not very obvious and
therefore all galaxies are used in the calculations of the separate bulge parameter distributions. The volume corrected distributions of the bulge $\mu_e$ and $r_e$ (Figs 15 and 16) show the same behavior as the disk parameters, i.e. constant distribution of the effective surface brightness and a steady decline of a factor of 50-100 per dex of the effective radius in the $K$ passband.

4.3 The bulge/disk relation

The chronology of the bulge and disk formation is a major issue and the relationships between bulge and disk parameters might give some insight in this matter. A strong correlation between bulge and disk parameters is expected if the bulge formed from the disk by secular evolution. A correlation might be expected in the hierarchical infall and small merger models producing bulges, because both bulge and disk originate from the same smaller components. In models where the bulge forms first and the disk forms later, there is no obvious reason for a bulge-disk correlation.

Comparing the $h_0$ with $\mu_e$ (Fig. 17) we see no correlation, except that the late-type spirals clearly stand out. This is most obvious in the $K$ passband. The correlation between $h$ and $r_e$ (Fig. 18) is only tight in the $K$ passband and not in the $B$ passband. Actually the correlation is becoming steadily tighter from the $B$ to the $K$ passband with correlation coefficients increasing from 0.6 in $B$ and $V$ to $\sim 0.75$ in $H$ and $I$ and $\sim 0.8$ in $H$ and $K$ passbands. The equation for the least squares fitted line is in the $K$ passband

$$\log(r_e^K) = 0.95 \log(h_e^K) - 0.86$$  \hspace{1cm} (1)

with a standard deviation of 0.17. The scalelength difference between bulge and disk thus is of order 10. The relation also holds for all the 1D fit techniques presented in Paper II (but less strongly) except for the case of an $r^{1/4}$ law bulge. In the case of 1D $r^{1/4}$ law bulge there is at best a weak trend (correlation coefficient 0.15). This relation could partly be produced by the fitting routine if the errors in both parameters are intrinsically correlated. The facts that this relation holds in the $K$ but not
Fig. 17. The central surface brightness of the disk versus the effective surface brightness of the bulge. Different symbols are used to denote the indicated morphological type ranges.

Fig. 18. The scalelength of the disks versus the effective radius of the bulge. Different symbols are used to denote the indicated morphological type ranges. The dashed line in the $K$ passband diagram gives the least squares fit relationship between both parameters.

Fig. 19. The galactic absorption corrected absolute magnitude of the disk versus that of bulge. Morphological types ranges are denoted by the indicated symbols.
in the $B$ passband and for both of the totally different 1D and 2D fitting techniques indicate that the correlation is intrinsic and not an artifact of the fit routines. The $\chi^2$ distribution around the solutions found by the fit routine also showed no correlation with the relationship between disk scalelength and bulge effective radius.

The absolute magnitudes of bulge and disk correlate well (Fig. 19). This is probably an example that large galaxies have more of everything, more bulge, more disk. The combined effect of good correlation between $r_e$ and $h$ and weak correlation between $M_d$ and $M_b$. Looking at the bulge-to-disk ratio as function of morphological type (Fig. 20), one sees that there is a correlation, but this correlation is less strong than for instance the one of $r_e$ with type (Fig. 10). The B/D ratio is on average higher in $K$ than in $B$; bulges are redder than disks. This partly explains why the correlations which involve bulge parameters are tighter in $K$. The bulge/disk decomposition is more easily performed when the bulge is relatively brighter.

Fig. 20 shows also that the selected galaxies are indeed disk dominated systems. The B/D ratios plotted here are much smaller than the ratios normally found in the literature. This is mainly due to the use of an exponential bulge. In Fig. 21 the results with an $r^{1/4}$ law bulge (Paper II) are shown. The B/D ratios are higher, but the scatter has increased and there still is no tight correlation with morphological type.

5 Discussion

In this section I will place the previously described relations in the context of the three topics of main interest: 1) Freeman’s law, 2) bivariate distributions and 3) the relation between Hubble classification and the structural parameters. I will conclude this section by confronting some galaxy formation and evolution theories with the newly found and some well known relationships. A combination of several models is probably needed to explain all aspects discussed here.

5.1 Freeman’s law

Since Freeman 1970 found disk central surface brightnesses to be constant among spiral galaxies, a number of explanations have been brought forward. In the introduction three expla-
nations were mentioned: 1) optically thick dust, 2) erroneous measuring of the disk parameters from the light profiles and 3) selection effects. For each of these possibilities I check if they are of importance for the current sample and whether they can explain Freeman’s result.

5.1.1 Optically thick dust

It has been suggested that optically thick dust could be the cause of Freeman’s law (Jura 1980; Valentijn 1990; Peletier et al. 1994). This is only a partial explanation, because it removes the inclination dependence from the law. To produce Freeman’s law in this way, galaxies must have the same surface brightness at $\tau = 1$ (where the typical surface brightness is produced in an optically thick system), which means the problem is only shifted from one part of the galaxy to another. One now has to explain why all galaxies have the same surface brightness at $\tau = 1$. Taking a constant dust-to-stellar light ratio will not produce a constant surface brightness. This is only the case if all dust is in front of the star light, but dust and stars are mixed in a galaxy and a fraction of stars to the near side is less obscured. The amount of extinction in a galaxy is not linearly dependent on the amount of dust present (see also Paper IV). Coupling the amount of dust and stars in galaxies can only reduce the scatter in $\mu_0$, but can never produce a constant $\mu_0$.

I have shown that applying the inclination correction of Eq. (1) reduces the scatter in the $\mu_0$ of the disk going from $C = 0$ to $C = 1$ (Table 1, indicating transparent behavior. The effect is small and the scatter is still dominated by the intrinsic differences in the brightnesses of the disks. One should realize that the disk parameters are largely determined by the outer regions of the galaxy. They probably do not reflect the optical thickness of the central regions. I note again that galaxies can behave optically thin in an inclination test, while in fact being optically thick.

The $K$ passband data should hardly be affected by dust extinction. Looking at Table 1 one can see that the standard deviation of $\mu_0$ is for the early-type galaxies smaller in the $K$ passband than in the $B$ passband, contrary to what is expected for optically thick dust. The increase in standard deviations for the later types can be explained by stellar population differences (Paper IV). I conclude that dust extinction is not a major effect in Freeman’s law, certainly not in the $K$ passband data used here.

5.1.2 Erroneous profile fitting

Kormendy (1977), Phillipps & Disney (1983) and Davies (1990) have argued that Freeman’s law results from fitting the exponential disks to light profiles without taking the bulge contribution to the profiles into account. To prove their point, they created model profiles with $r^{1/4}$ law bulges and exponential disks with a range of properties, to which exponential disks were fitted in a specified range. These models were able to reproduce Freeman’s “universal” central surface brightness value of $21.65$ $B$-mag arcsec$^{-2}$ with just a small scatter.

The parameter space explored in the models is not very physical according to current insights. Kormendy 1977 used B/D ratios of 1–120 for the low surface brightness systems and Phillipps & Disney 1983 assumed that bulges were so extended that they dominated the light profiles again at the $24.5$ $B$-mag arcsec$^{-2}$. Davies (1990) used a range of properties for the bulge parameters which are typical in samples of galaxies, which suffer from severe selection effects. He used a constant central surface brightness of the bulge to show that the central surface brightnesses of disks need not be constant. A change in $r_e/h$ ratio was used to produce a range in bulge-to-total light ratios ($BT$). I have shown that $\mu_e$ is not constant (Fig. 10) and that the $r_e/h$ ratio is nearly constant (Fig. 18). Even though these results where obtained with an exponential rather than a $r^{1/4}$ law bulge, a constant central brightness for bulges is excluded and a relationship between $r_e$ and $h$ might be expected.

I use the parameterization of Davies 1990 to show that erroneous bulge/disk decomposition is not a major factor in the Freeman law. Figure 22 was produced in the same way as Davies’ Fig. 6 by fitting exponential profiles in the range of 22 to 25.5 mag arcsec$^{-2}$ to model profiles, which were a combination of an $r^{1/4}$ law bulge and an exponential disk. In this figure the intrinsic $\mu_0$ of the model profile is compared with the central surface brightness of the fitted disk ($\mu_e$). The $r_e/h$ ratio was taken fixed to 0.4 and $\mu_e$ was adjusted to produce $BT$ ratios in the range from 0.05 to 0.75 in steps of 0.1 (contrary to Davies, who used a fixed $\mu_e$ and varied the $r_e/h$ ratio). The tendency to shift intrinsic bright disks to the observed value of 21.65 mag arcsec$^{-2}$ has disappeared. The low surface brightness disks have too bright $\mu_e$ values for their $\mu_0$, but these were clearly fitted in the curved part of the profile at the brighter end. From Fig. 22 one can conclude that it is unlikely that the high surface brightness disks were underestimated (even with using the “marking the disk” fit of Paper II). The situation for LSB systems is not as bad as it seems, because the B/D ratios are low for LSB galaxies and the curvature of the profiles can be readily seen. Choosing values for $r_e/h$ in the range from 0.1 to 1 hardly changes these results.

![Fig. 22. The variation of the measured extrapolated central surface brightness ($\mu_e$) with bulge-to-total light ratio ($BT$) ranging from 0.05 to 0.75, at a given model central surface brightness $\mu_0$ (see text).](image-url)
The central surface brightnesses of the galaxies were determined in Paper II with a full 2D decomposition technique, but also with the “marking the disk” technique and a few 1D decomposition techniques. If Freeman’s result was caused by his use of the “marking the disk” method, the central surface brightnesses obtained with this method should show large and systematic differences with the results of the other methods. In Paper II it was shown that making a proper decomposition of the profiles in a \( r^{1/4} \) law bulge and an exponential disk adds a scatter of at most 0.4 mag arcsec\(^{-2} \) to \( \mu_0 \) with respect to the “marking the disk” results. Assuming that about the same value would hold for Freeman’s sample, this would still result in a rather small range in \( \mu_0 \) for his sample. In Paper II it is furthermore argued that the 2D decomposition technique, using exponential light profiles for both disk and bulge, is more accurate and reduces the rms difference between the “marking the disk” method and the 2D fit to 0.3 mag arcsec\(^{-2} \).

Both the model decompositions using Davies’ method and the comparisons between different decomposition methods of real galaxies indicate that it is unlikely that Freeman’s results were caused by improper decompositions.

### 5.1.3 Selection effects

After taking selection effects into account, Fig. 8 shows that there is no such thing as a simple Freeman’s law for galaxies with scalelengths larger than 1 kpc. There seems to be a clear upper limit to the central surface brightness, which cannot be explained by selection effects. Even taking the apparent upper limit in absolute luminosity in Fig. 5 into account, there still should have been galaxies with \( \mu_0 \) brighter than 20 \( B \)-mag arcsec\(^{-2} \) (16 \( K \)-mag arcsec\(^{-2} \)) in the sample according to this figure. Figure 8 shows further that there is no strong decrease in the number of galaxies with lower surface brightness. The distribution becomes narrower if we exclude late-type spirals, but this exclusion can hardly be justified. Late-type spirals are in many respects no separate class of galaxies, but just a continuation of the trends set by the earlier type spirals. A clear example of such a trend is seen in Fig. 10.

In Paper II it was shown that there is at most \( \sim 0.30 \) mag arcsec\(^{-2} \) rms uncertainty in the central surface brightnesses. The uncertainties also showed no correlation with the surface brightness itself (Paper II, Fig. 4) and the results presented here can therefore not be the results of measurement errors. The central surface brightness distribution of Fig. 8 changes in some details if one of the other fit techniques of Paper II is used, but the general trend remains unchanged. The same holds true when the distances of the galaxies are calculated with other flow models.

Sample selections are influenced by both \( \mu_0 \) and \( h \) and therefore the most important distribution for disk dominated galaxies is the bivariate distribution in the \( (\mu_0, h) \)-plane. These two parameters describe a large fraction of the light of disk dominated galaxies and to derive this distribution one needs distances. To derive the distribution of \( \mu_0 \) of sample of galaxies without knowing the distance to the galaxies, one must assume that the distribution of \( \mu_0 \) is not correlated to for instance \( h \) or \( M \) (Davies et al. 1994; McGaugh et al. 1995).

The total \( \mu_0 \) distribution can only be calculated in this statistical way if the \( \mu_0 \) distribution at each \( h \) or at each \( M \) has the same shape. Figure 7 shows that this is probably not the case for \( h \) and Figure 23 shows the same for \( M \). The statistical methods can at best only be used to get an impression of the \( \mu_0 \) distribution.

### 5.2 Bivariate distributions

The reason for our limited knowledge of low surface brightness galaxies is clearly indicated by the selection limits in Fig. 5. The use of catalogs like the UGC prevents galaxies with central surface brightness fainter than \( \sim 25 \) \( B \)-mag arcsec\(^{-2} \) from being included in a sample, independent whether the sample is diameter or magnitude selected. Only the use of deeper photographic plates enabling a selection at fainter isophotes (Schombert et al. 1992) or deep CCD surveys will result in samples with a larger number of low surface brightness galaxies. A galaxy like Malin I (Impey & Bothun 1989), with \( \mu_0 \approx 26.5 \) \( B \)-mag arcsec\(^{-2} \) and \( h \approx 55 \) kpc, is not found in conventional catalogs, even though it has an integrated magnitude comparable to M101 and a \( \sim 10 \) times as a large scalelength as M101. Figure 7 indicates that galaxies like Malin I are probably not very numerous in the local universe, but this cannot be said of galaxies with \( \mu_0 > 24 \) \( B \)-mag arcsec\(^{-2} \) and \( h \approx 1 \) kpc. There is a clear need for deeper local surveys, especially in the near-IR.

Few bivariate distributions have appeared in the literature which can be used in comparisons with the current results. Van der Kruit (1987) calculated a bivariate distribution of spiral galaxies in the \( (\mu_0, h) \)-plane. His distribution shows similar features as the distribution presented here. The distribution has an upper limit in central surface brightness at about 21 \( J \)-mag arcsec\(^{-2} \)(photographic \( J \)-passband, which is similar to the Johnson \( B \)-passband) and an exponentially declining density distribution with scalelength.

Another comparison can be made with the bivariate distribution of van der Kruit (1989). This distribution was constructed in exactly the same way as Fig. 7, with only a modification to Eq. (3) to include the effects of an additional magnitude selection limit. The central surface brightnesses were convolved with a Gaussian of 0.3 mag arcsec\(^{-2} \) to incorporate the effects of calibration uncertainty. This explains the smoothness of the distribution. The galaxies of type later than \( T = 5 \) were excluded and a Hubble constant of 75 km s\(^{-1} \) Mpc\(^{-1} \) was used. The same upper limits in absolute magnitude and central surface brightness can be seen as in Fig. 5. There is only one galaxy with \( \mu_0 \) brighter than 18 \( R \)-mag arcsec\(^{-2} \) at 0.7(100/75) kpc. The general trend in the bivariate distribution of van der Kruit agrees quite well with Fig. 7, with equal probabilities along lines of equal absolute luminosity.

The luminosity function of galaxies is a tool often used to investigate galaxy evolution on cosmological time scales. This makes sense as the total luminosity of a galaxy seems, also by use of the TF-relation, related to the total mass of a galaxy. In determining the local LF one has sometimes failed to notice that galaxies are extended and that for selection correction
one should not treat them as point sources. This can lead to a change in the slope of the LF as function of redshift and results in the faint blue galaxy problem (McGaugh 1994).

Another disadvantage of the one-dimensional LF is that each luminosity bin contains galaxies with totally different surface brightnesses and scalelengths (Fig. 5). The physical processes in disk galaxies seem to be more related to surface brightness than to total mass (see also Paper IV). Therefore, to investigate the distributions related to both the total mass and the surface brightness, the luminosity function has been divided in several central surface brightness bins in Fig. 23. The absolute magnitudes were calculated from the values given in Paper I. Figure 23 shows the bivariate distribution of a local sample of galaxies and can be useful as a reference for high redshift samples observed with the Hubble Space Telescope.

Figure 23 shows that the LF is more or less the same for all central surface brightness bins in the $B$ passband. In the $K$ passband something becomes apparent which was already hinted at in the $B$ passband. The LF for fainter central surface brightnesses is lower and/or shifted to lower absolute luminosities. Figure 23 is somewhere in between both options presented in Fig. 3 of McGaugh (1994), which means that both the shape of the LF and its normalization depend on the bivariate distribution of $M$ and $\mu_0$. Unfortunately, this data is too scarce to make a firm quantitative statement, but it is clear that further attention should be given to “the LF”.

It is also interesting to know what type of galaxies provides most of the total luminosity in the local universe. In order to calculate this, the number density value of each galaxy used in Fig. 23 was given an additional weight depending on its absolute luminosity, which results in Fig. 24. This figure shows...
the total luminosity one expects to find in a random Mpc³ from
galaxies in the indicated bins of (μ₀, M). The luminosities are
expressed in solar luminosities per passband, calibrated using
the absolute solar luminosity values of Worthey (1994).

The distribution in the B passband is remarkably flat,
almost all bins that contain galaxies are equal to within 1.5
order of magnitude. In the K passband distribution there is
more structure, as most of the K passband light in the local
universe comes from higher surface brightness galaxies. Due to
the scarceness of the data, this figure is again more of qualitative
than of quantitative interest.

5.3 Hubble classification

The Hubble sequence is one of the basic ingredients of galaxy
formation and evolution schemes, even though the underlying
physical processes are only partly understood. In this section
I describe the consequences of some of the relations between
the structural parameters and Hubble type as presented here.

Figure 20 showed that B/D ratio cannot be used to deter-
mine the Hubble type of face-on systems, which means that
the classification of edge-on systems is different from that of
face-on systems. Furthermore, the B/D ratios seem to be quite
small (< 0.5), even in the K passband where the differences in
color (and M/L) due to stellar population effects between disk
and bulge are minimized. The B/D ratios are larger when r¹/⁴
law bulges are used. The difference between exponential and
r¹/⁴ law bulges in terms of generalized exponential profiles
is extensively discussed in Paper II. Young & Curry (1994)
showed that for ellipticals and dwarf ellipticals there is a trend
in profile shape with luminosity. The brighter ellipticals have
r¹/⁴ like profiles, while fainter (dwarf) ellipticals have more
exponential like profiles. If this is also true for bulges, we might
expect early-type spirals to have more centrally peaked bulge
profiles than later type spirals. In Paper II it was found that r¹/²
law bulges gave smaller χ² residuals for T<3 than exponential
bulges. This picture conflicts with the scale independence of
Hubble type as seen in the K passband data of Fig. 18. Within
one Hubble type, a range in integrated bulge luminosities exists,
which should result in different profiles when the model of
Young & Curry (1994) is applied to bulges. There is a weak
indication for this trend, because the galaxies with the brightest
bulges are slightly better fitted by the r¹/² law bulges than by
the exponential bulges. The r¹/⁴ law bulges never give the
smallest residuals, not even for the most luminous bulges.

Using an exponential bulge, an important relation is found,
which is not apparent if an r¹/⁴ law bulge is used: h correlates
with rₑ, but these parameters do not correlate with Hubble type
(Fig. 18). This is an important relation as it makes the Hubble
sequence scale free. Each Hubble type comes in a range of
different sizes, both in terms of diameter and total luminosity
It is an example of larger (in the sense of scale size, not mass)
galaxies having more of everything, larger disk, larger bulge.
Due to this correlation the scale parameters cancel each other
out in calculating B/D ratios, which means that a plot of μₑ−
μ₀ (∝ Σₑ/Σ₀, important in density wave models) versus type
looks like Fig. 20 with a different scaling. Consequently μₑ−μ₀
is also a bad diagnostic for Hubble type.

The relationship between Hubble type and μₑ (Fig. 10)
holds, independent of bulge profile function used. In fact the
relation holds if one just fits a line to the central region of the
profile and uses the true central surface brightnesses, because
the bulge generally dominates the luminosity in this central
region. One might wonder if in classifying galaxies one has
mainly looked at the surface brightness of the bulge and not at
the B/D ratio. For the earlier systems this is harder to accept;
the central regions are in general overexposed on photographic
plates used for classifying. It is instructive to know that S0 gal-
axies do not fit in this relation. The central surface brightnesses
of S0 galaxies range from ~17.7 to ~19 B-mag arcsec⁻²,
estimated from the data of Kormendy (1977) and Peletier et
al. (1990). This translates to effective surface brightnesses in
the range of 19.5-21 B-mag arcsec⁻², in accordance with the
two S0 galaxies in the current sample, but significantly below
the trend of the rest of the spiral galaxies (Fig. 10).

If B/D ratio is such a bad diagnostic for Hubble type,
we are left according to Sandage (1961) with only two other
classification discriminators: 1) the spiral arm structure, 2) the
pitch angle of the arms. The two remaining criteria indicate that
the Hubble sequence should be explained in terms of spiral arm
appearance, even though the second criterion might also be in
doubt, as measurements by Kennicutt (1981) showed that pitch
angle has no tight correlation with morphological type.

The main theory on spiral structure is the spiral density
wave theory (Lin & Shu 1964; Roberts et al. 1975). The fact
that Hubble type is a scale free quantity fits into this theory.
When the bulge and the disk scale with the same amount, the
shape of the rotation curve stays the same, only its amplitude
changes. This means that the shape of the resonances also
scale along with the scalelength changes. If bulge and disk
scalelengths are correlated, the shape of the rotation curve is
fully determined by the relative brightness of the bulge and disk
component. Therefore it is harder to understand in the density
wave theory why Hubble type does not correlate tightly with
B/D ratio (Fig. 20), or to be more precise μₑ−μ₀. The strength
and the pitch angle of the density wave gets modified by the
ratio of mass that participates in the density wave to the mass
that does not. If B/D ratios are so small that they hardly could
affect the density wave (Fig. 20) and if on top of that the B/D
ratios and μₑ−μ₀ values are only weakly correlated with Hubble
type, it seems that some modifications to the standard density
wave model are needed. Maybe a connection between the μₑ
of the bulge and the distribution of dark matter can solve this
problem.

5.4 Galaxy formation and evolution models

A number of new observations and relations have been pre-
sentated here, which can be compared with the predictions of
existing galaxy formation and evolution models, e.g.:
1) the upper limit to disk central surface brightness.
2) the nearly constant Φ(μₑ).
3) the correlation of the bulge central surface brightness with
4) the scale independence of Hubble type.

Most formation and evolution models were designed to explain other observations. One can think of the exponential radial light distribution of disk galaxies, the TF-relation, the density-morphology relation (Dressler 1980; Postman & Geller 1984) and the fact that disk scalelength is constant as function of radius (Shaw & Gilmore 1990). Also the relative fractions of different Hubble types needs to be explained.

The models in the literature are in general scale free. This means that physical limits in surface brightness or total luminosity as presented here are often not discussed. As a consequence the following discussion will be qualitative, not quantitative. The Freeman value of 21.65 \( B \)-mag arcsec\(^{-2}\) is often assumed for the central surface brightness in these evolution models, but as shown here this is an invalid simplification.

The chronology of bulge and disk formation is still a major issue. The disk galaxy evolution models can to first order be divided into three categories: those that form the disk first, those that form the bulge first, and hierarchical clustering models in which both are formed together by accreting smaller clumps of proto-galaxies. These models can partly be mixed at the different stages of galaxy evolution. Figure 18 shows that bulge and disk scalelengths are correlated. In models where the bulge forms first and then the disk, it is hard to understand how a small dynamically hot bulge can influence the disk scalelength. In models where the bulge forms from the disk, a natural correlation between their scalelengths is expected. The situation is reversed for S0 galaxies, because in these galaxies the bulge is much larger than the disk.

In recent years theories on galaxy formation start making use of the developments in the study of large-scale structure within the framework of the standard cosmological models, such as Cold Dark Matter (CDM) models (Blumenthal et al. 1984). In the standard cosmological models a galaxy principally forms when its matter dynamically decouples from the main Hubble flow. The observations presented here concern the stellar component of galaxies and the main obstacle to translate the predictions of the CDM models into these observations are the poorly known star formation mechanisms. There are some qualitative scenarios relating initial conditions to morphological type (e.g. Lake & Carlberg 1988; Zaritsky 1993) and therefore to the stellar component. In these scenarios the large potential well, which will form the final galaxy, contains smaller density fluctuations. In some of these subclumps enhanced star formation may occur, triggered by neighboring clumps or galaxies. The clumps with enhanced star formation violently relax in the main potential well to become the bulge and the halo, while the other clumps dissipate and form the disk. Such scenarios can hardly be compared with the quantitative descriptions of the bivariate distribution (most notably the upper limit in \( \mu_0 \)) and the correlation between bulge and disk scalelength presented here as long as there are no quantitative descriptions of the star formation processes involved.

The secular evolution models in which the bulge is formed from the disk have more predictive power. In these models the disk is formed first by a dissipational collapse of the initial gas cloud after getting its angular momentum from tidal torques from neighboring galaxies. In the following some secular evolution models are described. All of them could play a role after an initial collapse as described by the CDM models has occurred.

One of the models that has successfully explained the nature of exponential disks is the viscous evolution model (Lin & Pringle 1987; Yoshii & Sommer-Larsen 1989). In this model the angular momentum in the disk is redistributed by the viscosity of the gas, while star formation occurs on the same time scale. Saio & Yoshii (1990) showed that such a model automatically develops, next to the exponential disk, a bulge with properties only depending on the time scale of star formation relative to the time scale of viscosity and on the total angular momentum. This model could produce galaxies with correlated bulge and disk scalelengths. There is no obvious reason why there should be an upper limit to the surface brightness in these models.

The secular evolution model (Kormendy 1993 and references therein), in which small bulges form from the bar instability, can explain a number of the observations described here. In this model disk gas is transported to the center by a bar or oval distortion. The disk stars in the central region are heated in vertical direction by resonant scattering off of the bar instability to become a “bulge-like” component (Combes & Sanders 1981; Pfenniger & Friedli 1991). This model will naturally develop an exponential bulge, where only a limited range in bulge-to-disk scalelengths is possible. Furthermore, the bar forms when the disk surface density is too high for its velocity dispersion (see e.g. Binney & Tremaine 1987), which might explain the upper limit observed for surface brightnesses. The density-morphology relation is in this model partly explained by the higher chance of dynamical interaction in denser environments, which triggers the bar formation (Elmegreen et al. 1990).

A model to explain the Hubble sequence is the model of bulge formation by a central starburst, by Sofue & Habe (1992). This model has links with the previous two models. Instead of a bar, galactic winds driven by starbursts are used to produce the dynamical hotter bulge component from the disk component. Tidal interactions are known to produce starbursts, therefore more interactions give rise to larger bulges, thus explaining the density-morphology relation. This model might explain the upper limit in central surface brightness observed for the sample galaxies. When a critical gas density is reached, a burst of star formation will occur and all remaining gas will be expelled or put in orbits with a much higher vertical component. The “chimneys” of ionized gas observed in some edge-on galaxies (Rand et al. 1990) indicate that large amounts of gas can be driven out off the disk plane.

6 Conclusions

The statistics of the fundamental parameters of 86 spiral galaxies have been studied in the optical and the near-IR. The use of the near-IR \( K \) passband enabled for the first time determination of these parameters without being hampered by the effects of dust and differences in stellar populations.
Volume density distributions with respect to the fundamental parameters were made, which was possible due to the careful selection of the sample. The main conclusions are as follows:

- Freeman's law of a preferred disk central surface brightness value needs a modification. Although there seems to be a clear upper limit to the central surface brightnesses of galaxies, there is no clear limit at the faint end of the $\mu_0$ distribution. The number of galaxies in a volume with a certain $\mu_0$ is only slowly declining function of $\mu_0$.
- The bulge and disk scalelengths are correlated, parameterized by $\log(r_K^b) = 0.95 \log(h_K^b) - 0.86$. This correlation suggests that the formation of the bulge and the disk is coupled.
- The Hubble classification is related to the surface brightness of spiral galaxies. However, the relations are in general not very tight and can therefore not be turned around to give morphological classification. The B/D ratio is not a good indication of Hubble type (Fig. 20 and 21). The best relation with Hubble type found in this study is the one with $\mu_e$ (Fig. 10). The physical interpretation is difficult, because cause and effect are hard to separate.
- Hubble type is a scale size independent parameter, but not a total luminosity independent parameter of a galaxy. Therefore, it would be better to divide by scale size instead of luminosity to derive scale independent parameters of galaxies in comparisons. The suggestion that Hubble type is mainly driven by total mass (Zaritsky 1993) seems an oversimplification. To carry this point a bit further, it is probably better to separate the determination of the LF of galaxies into bins which are related to the effective surface brightness (like Fig. 23) than into bins which are related to Hubble type.

Larger samples are needed to enable the parameterization of the bivariate distributions. There is especially need for accurate surface photometry of a large sample of galaxies, selected from deep photographic plates providing isophotal diameters at very faint levels.

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